EXPLORE
COMPUTING
with the
TRS-80

and COMMON SENSE

by Richard V. Andree & Josephine P. Andree

YOUR AUTHORS WISH TO EXPRESS OUR SINCERE THANKS TO THE
MANY TEACHERS WHO HAVE HELPED IN THE DEVELOPMENT OF THIS BOOK,
AND TO ENCOURAGE YOU TO CORRESPOND WITH US CONCERNING YOUR
CORRECTIONS AND SUGGESTIONS ON WHAT YOU WOULD CONSIDER PARTIC-
ULARLY HELPFUL IN AN INSTRUCTOR'S MANUAL.

OUR "COVER GIRL" IS SUZANNE M. ANDREE, AN ELEMENTARY
MATHEMATICS-SCIENCE TEACHER WHO USES COMPUTERS IN HER

TO THE READER

The first 100 pages carefully lead you from "watching the computer count fast" to where you have enough skill and confidence to write your own computer programs for creating random art, solving equations, or exploring microRESEARCH problems that interest you.

If you are a complete novice, start with the introduction.

If you are a beginner, start with Lesson 1. Work as many practice problems as you need.

If you are a bit more sophisticated, read the summary at the end of Lesson 1, work a few problems from Practice Session 1. Then continue with Lesson 2.

If you can already program in BASIC, read the summaries at the end of Lessons 1 & 2, work a few problems, and continue with Lesson 3.

Lesson 4 introduces graphics, computer assisted art and some of the special goodies available on the TRS-80.

Lesson 5 is devoted to solving problems that are easy to understand, but which would be very hard to solve without the assistance of a computer.

PART II

The first five Lessons will teach you most of what you need to know to use your computer in solving real problems.

The nature of Lessons 6 to 17 is different. These lessons are devoted to special topics you may or may not need to learn about, depending upon your own particular interests. Lessons 6 to 17 may be read in any order, or ignored entirely until needed. Consult the table of contents for topics covered.

The important thing is to HAVE FUN with your TRS-80. Remember, today's micro-computers are more versatile and more powerful than computers which cost 100 times as much, 20 years ago. Whether you purchased your TRS-80 as a playtoy or a serious tool, we hope you will soon be using it for both.

TO THE INSTRUCTOR

This book is designed to be used by students for self-instruction
(with or without a teacher's assistance). It has been successfully used
by junior and senior high school students, secretaries, college students,
6th grade math classes, as well as a group of busy doctors and business
people. Numerous examples are developed step by step to demonstrate the
thinking involved. Since computing is a skill that must be learned by
practice, a variety of problems are included. As with any acquired skill,
a good coach can keep the neophyte interested and help develop good hab-
its. Computing can and should be FUN. Students are fascinated by com-
puters and will be responsive to your teaching.

Most readers are familiar with whole-number arithmetic, but may not
have investigated the many tantalizing arithmetic puzzles that are easy
to understand but hard to solve, unless a computer is available. A num-
ber of our problems come from the area of Number Theory. Problems like

1. Find squares like $(11)^2 = 121$ or $(264)^2 = 69696$ whose digits
    read the same forward as backward (palindromic squares).

2. Find squares such as $(35853)^2 = 1285437609$ in which all ten
    digits are present in the square.

are not difficult to program once the rudiments are learned (Lessons 1,
2,3). Each reader is encouraged to use common sense to write more effec-
tive programs. Lessons 4,5,7,9,13,14, and 17 provide numerous interest-
catching problems from non-mathematical areas. We hope each reader will
discover new interests and power as his programming skills develop.

Readers are encouraged to develop orderly (top-down, structured) pro-
gramming habits without being preached at. Your assistance is sought
in helping students develop this vital skill.

Lessons 5 and 17 present 75 micro-research problems that are easy to
understand. Micro-research problems demand thoughtful program design to
produce results with a reasonable investment of computer time. All have
been successfully programmed by students with no more than 9th grade
mathematical skill. In many cases, the programs "develop" as do the ex-
amples given in the text, starting first with a brute-force attack which
could require weeks of computer time, and then gradually, as the early
results are available for examination, refinements in program design
(early trimming of logic trees) suggest themselves. Eventually, many
students devise programs that produce more results in 5 minutes than the
original program could produce in five days.

Appreciation and understanding of the value of skillful computing
techniques are developed as needed, through experience.

The material has been used with Models I, II, & III TRS-80 micro-computers to teach introductory computing concepts to several different groups, including students in grades 5 through 14, adult neophytes having no experience on computers and little math beyond the 8th grade, secretaries and secretaries-in-training, a group of busy doctors, bankers and business people, and casual adult and child visitors at a museum. Each contributed to the presentation contained herein. Every vital concept is presented more than once, in different environments, to aid the student's comprehension. The important thing is to be sure readers also actively program computers, not passively read the text. Passive reading, like a spectator sport, does not develop skills. Encourage students to really program various kinds of problems.

With the exception of Lesson 4 ART & GRAPHICS and a few special goodies available only on the TRS-80 Level II BASIC, most of the material presented here is applicable to any computer using BASIC.

The first five lessons form a unit. Lessons 6 through 17 may be taken up in any order or ignored, as individual interests suggest.

Our experience suggests having two or three students per computer produces better results than one person per computer--particularly in the early stages.

Your authors welcome criticism, suggestions and correspondence.

Richard V. Andree & Josephine P. Andree

The University of Oklahoma

Norman, Oklahoma   73019

ACKNOWLEDGEMENTS

CONTENTS

If you have never used a computer before, this introduction was written especially to help you gain familiarity with the keyboard, the screen and the use of BREAK , CLEAR , and ENTER keys. Your computer will do arithmetic and display messages. The main purpose of the introduction is to lift you from neophyte to beginner.

Beginners start here. A four-step program shows off your computer's speed as well as its versatility. Program instructions, PRINT variations, and additional keyboard commands are introduced.

Learn by programming the computer to create tables of squares, cubes, etc., for varying ranges and different step sizes. You are still not an expert, but you are well started.

You and your computer will solve problems that would be difficult, if not impossible, to solve without a computer. Nasty equations are easy to understand and to solve with the help of a simple computer program.

Although this lesson is mostly fun, the ideas developed help design
computer games and present graphical material in a dynamic format that
is easy to understand. Lots of room for fun and self-expression here.

Surprising as it may seem, you already have enough computing-know-how
to solve problems that would be too difficult to consider without com-
puter assistance. A bit of eighth-grade mathematics, common sense, and
your computer are all that is needed to investigate problems that would
intimidate a college student who did not have access to a computer or
calculator.

A review of the contents of the rest of the book—which may be taken up
in any order desired, or ignored, whichever serves your needs best.

As you develop longer programs you will wish to save the program on
magnetic tape using the cassette player that came with your computer.
Tape cassettes are a standard way of trading programs with other com-
puter owners. Cassettes are used for storing data and results as well
as programs.

Everyone loves to play computer games. Use your own ingenuity to create
new game programs and to improve programs written by others.

A serious programmer needs to know more about the powerful EDIT instruc-
tions available on your computer. This is where you will find answers.

College courses are given on SIMULATION, but the basic philosophy
involved is easy to understand and not difficult to program. This
introduction removes much of the mystery and enables you to program
your own simulations and computer games.

LESSON 16   ADDITIONAL BASIC   184

Additional BASIC instructions are available if and when you need them.
Turn here if you need to do something we haven't discussed.

LESSON 17   MORE micro-RESEARCH PROBLEMS   188

Lesson 5 presented 17 micro-RESEARCH PROBLEMS you could explore with
the computing expertise available then. Here we discuss programming
practice, design and improvement of programs using easy-to-follow
examples. Lesson 17 then completes our collection of seventy-five (75)
choice micro-RESEARCH problems, each worthy of exploration, expansion,
and investigation. This collection contains a wide variety of proposals
including several arithmetical explorations, Haiku poetry, logical
decisions, calendar problems, dart and target games, loan and interest
programs, treasure hunt games, caricatures, a speech timer, puzzles,
recursive functions, polygonal maps, medical emergency prompter, num-
ber theory, lattice problems, magic prime squares, graphs, monkeys at
typewriters, amicable and sociable numbers, dance partners, and the
notorious eight queens problem. They are here for your entertainment
and education. Enjoy them.

A FINAL WORD from your authors   214

INDEX   215
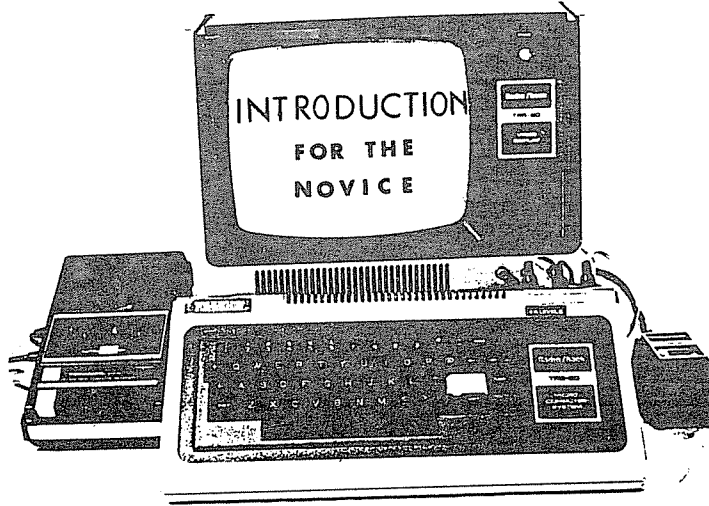
We assume you have no knowledge of computing, but are interested in learning. The way to learn computing is to compute!

Just as in golf or bowling, it is more helpful to read about computing *after* you have tried your hand at the game enough to have a feel for its rudimentary swing. So let's begin.

Let's start by using the computer as if it were a pocket calculator. It is a waste of the computer's power, but it will help you become familiar with the keyboard. Have someone turn it on for you. Depress the white ENTER key a couple of times.
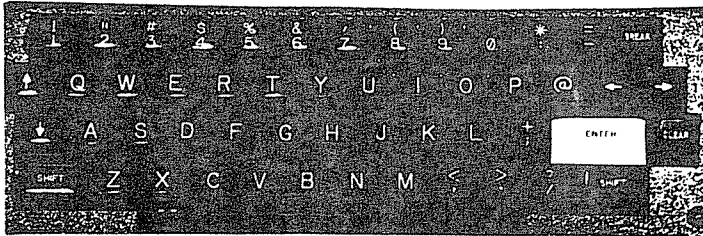
The keyboard is similar to a typewriter. If you wish to use a symbol like = or + from the top row of the key, depress the SHIFT key. The computer automatically types capital letters, as shown on the keys, so do *not* use the shift key unless you wish a symbol from the top half of a key.

Don't try to "read" this introduction—the way to learn computing is to compute, so wait until you have a TRS-80 at your fingertips before continuing.

Depress the following keys:

1. BREAK

(This interrupts the computer in case it was doing something.)

2. ENTER

(This is the key you will use to send your typed message to the computer.)

3. Now type: PRINT 3+5+9-2 ENTER

O, HELLO. DID YOU WANT TO TALK TO ...?

The computer should respond with the sum 15.

Your CRT-display will now show:

```
READY
>PRINT 3+5+9-2
15
READY
>_
```
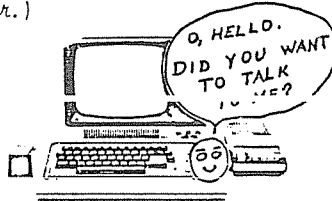
The > symbol indicates the computer is ready to receive your typed instruction.

The - shows where the next typed character will appear.

Should you wish to clear the screen, depress the CLEAR button, but there is no need to do so unless you wish it cleared. You may also type CLS ENTER to CLear the Screen.

What will the answer be if you type ?

PRINT 4*9 - 2 ENTER

*Most computers use the symbols * for multiply and / for divide.*

Try it and see.

Forecast the result of each of the following, and then try them out on your TRS-80.

PRINT 4/3 ENTER
PRINT 9*5 ENTER
PRINT 360*18 ENTER
PRINT 1/2 + Ø.6 ENTER

Note Ø is zero on the top row. 0 is the letter "oh" which is on the second row. Your computer will be unhappy if you confuse them. Try it once and see for yourself.

Let's try some hard problems

PRINT 774*817 ENTER
PRINT 79276/511Ø17 ENTER
PRINT 317*1.41421 - 246*3.14159 ENTER

*If you type 774*817 ENTER without PRINT, the computer will do the arithmetic but will not print or store the result.*

The `◁══` key is another important key for those of us who occasion-
ally mistype. It will backspace and erase one character at a time. If
you hold down the `SHIFT` key while you depress `◁══` the entire line
you typed will be erased. Try it and see for yourself.

Type

PRINT "HELLO " `ENTER`
                    Note the " " marks.
                    What happened?

Type

PRINT "SALUTATIONS " `ENTER`

PRINT "MY NAME IS *your name* " `ENTER`

Now let's try something different. This time we shall put a "line
number" in front of each statement. First type   NEW   `ENTER`

Then type:

100 PRINT "HELLO " `ENTER`
110 PRINT "SALUTATIONS " `ENTER`
120 GOTO 100 `ENTER`

That isn't progress, or is it? Well, nothing happened. Did it? Yes, it
did, but it happened <u>inside</u> the computer, where we can't see it, not on
the screen.

Type

                       The computer should list the three
                       statements you typed in. Check them
LIST `ENTER`   *to be sure they were properly entered*
                       with a space after each number
                       and " " in the proper places.

Next type

RUN `ENTER`

After a bit, depress the `BREAK` key to get control again.
Depress the `CLEAR` button.
Type LIST `ENTER`
and the computer should respond by displaying:

100 PRINT "HELLO "
110 PRINT "SALUTATIONS "
120 GOTO 100
READY
>_

It is perfectly reasonable to change these instructions by retyping them.

Type

    1∅∅ PRINT "HELLO *your name*";   [ENTER]   *Note the " " and the ;.*

    11∅ [ENTER]   *This erases the instruction in 11∅*
                       as you will see when you type LIST [ENTER] .

Now type

    LIST   [ENTER]

The screen should show

    1∅∅ PRINT "HELLO *your name*";

    12∅ GOTO 1∅∅

Note that instruction 1∅∅ was changed and instruction 11∅ was deleted,
but instruction 12∅ is unchanged.

Now type

    RUN   [ENTER]

    Let us write another program. Begin by depressing [BREAK] to get the
computer's attention. Then type:

    NEW   [ENTER]
    1∅∅ X = 1980   [ENTER]
    11∅ Y = 365   [ENTER]
    12∅ Z = X*Y   [ENTER]
    13∅ PRINT X; Y, Z   [ENTER]
    14∅ GOTO 1∅∅   [ENTER]
    RUN   [ENTER]

The above program sets X=1980, then sets Y=365, then forms the product of
the values of X and Y and stores it in a location called Z. Instruction
13∅ then prints out the values of the numbers stored in locations X, Y,
and Z. Instruction 14∅ then sends the computer back to instruction 1∅∅,
which repeats the entire process.

    Depress the [BREAK] key to stop the computer.

4

SUMMARY OF INTRODUCTION

Let's see what you have learned thus far...

To get the computer's attention, depress ⬚BREAK key.

Once you get the computer's attention, which it indicates by displaying

```
  ╭─────────────────╮
  │                 │
  │  READY          │
  │  >_             │
  │                 │
  │                 │
  ╰─────────────────╯
```

on the CRT (TV screen), you can work arithmetic problems involving +, -, *, / (add, subtract, multiply, divide) by typing PRINT followed by the arithmetic problem.

When the problem is displayed the way you wish it, depress ⬚ENTER to send it to the computer. Remember, nothing goes to the computer until you depress ⬚ENTER .

Oh, yes. If you make an error in typing, just depress the ⬚<═ key to backspace and erase. You can do it anytime before you depress the ⬚ENTER key.

To clear the screen, depress the ⬚CLEAR key, followed by ⬚ENTER if the >_ symbol is absent, or type CLS ⬚ENTER .

You need to be careful about confusing zero, 0, and the letter oh, O. Also, 1 and L are distinct.

You have written a set of instructions to the computer, and the computer has followed these instructions.

Now that you no longer fear the TRS-80, let's begin our lessons.

5

LESSON 1
BEGINNING
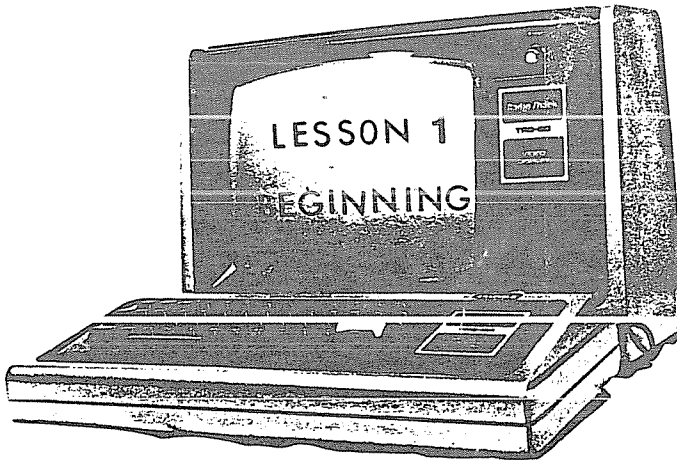
So you don't know anything about computers or computing. Well, this won't make you an expert, but let's see what we can do.

Someone has connected and assembled the TRS-80 and turned it on for you. The screen says something on it. I don't know what because I don't know what was done last.

If your TRS-80 has just been turned on, the screen may display

MEMORY SIZE?
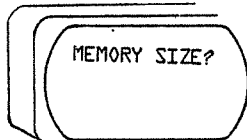
The TRS-80 can set aside memory for programs written in languages other than BASIC, but we shall not do so.

If so, just depress the white ENTER key. The screen will then display an additional message, which should end with

READY
>_

The word READY is your clue that the computer is ready to accept instructions from you.

To clear up everything...

Depress the |BREAK| key (upper right).
The computer will add

```
READY
>_
```
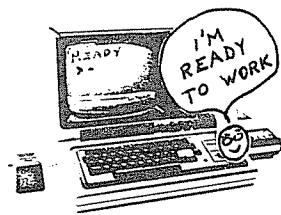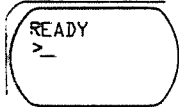
below whatever was on the screen. Now you have its attention. The >-
symbol below READY means the TRS-80 is waiting for you to type in an
instruction.

Type NEW and depress the white |ENTER| key.

Everything on the screen disappears—also any program inside disap-
pears too—the slate is wiped clean for you, and the computer displays

```
READY
>_
```
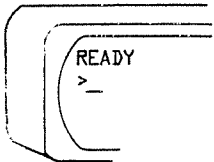
*NOTE: Nothing goes from the screen
to the computer until you depress
the |ENTER| key.*

Now type

21Ø X = 1 |ENTER|

The screen should now show
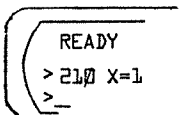
If you mistype, just use the |←|
*button to backspace and erase--or
depress* |BREAK| *and start the en-
tire line over.*

```
READY
> 21Ø X=1
>_
```

The symbol >_ indicates the TRS-80 is ready to accept another instruction,
so type

22Ø PRINT X; |ENTER| *Don't forget the ;*

The screen now shows

```
READY
> 21Ø X=1
> 22Ø PRINT X;
> _
```

7
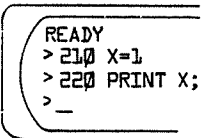
Continuing, type

>   23Ø X = X + 1   ENTER
         24Ø GOTO 22Ø   ENTER   *Note that GOTO is one word.*

To make sure the program you typed is really there, type

         LIST   ENTER

Your program will then be listed below whatever is already on the screen.

```
READY
>21Ø X=1
>22Ø PRINT X;
>23Ø X=X+1
>24Ø GOTO 22Ø
>LIST
21Ø X=1
22Ø PRINT X;
23Ø X=X+1
24Ø GOTO 22Ø
>_
```
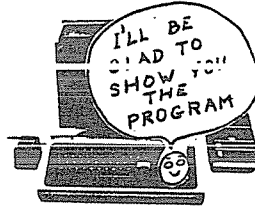

Note the absence of the > symbols on the LISTed program, indicating the computer typed it to you.

Now you are ready to run your first numerical program. Type RUN   ENTER   and watch what happens.

You probably have a Level II BASIC, but if not, read the right-hand column instead of the left-hand column below.

Level II BASIC

If you hold down SHIFT and depress @ , the program will halt until you depress another key (without SHIFT). Try it a few times and see.

Level I BASIC

If you depress the ↑ key on left edge of the keyboard, the program will halt whatever it is doing, as long as you keep the ↑ key depressed, but will continue whenever you let it up. Try it a few times and see.

In either case, if you wish an extended halt, depress the BREAK key. (Do so now.) Not only does the TRS-80 stop, but it even tells you which instruction it was executing when you depressed BREAK . If you want the program to continue, simply type: CONT   ENTER and it will continue.

Note that in Level I BASIC, if the program is typing a number, say 425 or 438, and runs out of line, it prints part of the number at the end of one line and the rest of the number at the beginning of the next line. The output is difficult to read, but the data is printed quickly and the screen displays a lot of data at once.

Level II BASIC is a bit more sophisticated and will leave the end of the line blank, rather than break a number, but it still changes the column spacing as the number of digits in the number changes in order to get as much output on a line as is convenient. The ; at the end of
  22Ø PRINT X;
is what controls the "packed spacing" of this output.

    Depress [BREAK]

    Type RUN [ENTER]

    to see what is meant.

Let's look at our program again.

Depress the [BREAK] key and the [CLEAR] key.

Then type LIST [ENTER]

The computer should respond.

```
>LIST
 21Ø X=1
 22Ø PRINT X ;
 23Ø X=X+1
 24Ø GOTO 22Ø
 READY
>_
```
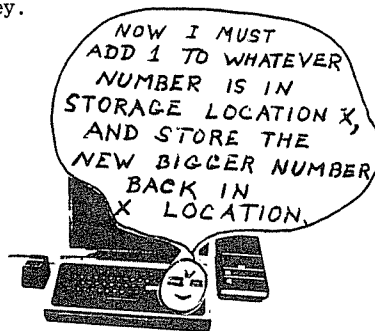
    Instruction 23Ø X=X+1 looks odd. If it were a mathematical equation, it would have no solution. It is <u>not</u> a mathematical equation; it is an in- struction to a computer.

X=X+1   This instructs the computer to "Take the number in the storage location called X, add 1 to it, store the new result back into the storage location called X."

X=X+1 could be more reasonably written as X←X+1. However, it is almost universal computer practice to use = rather than ← , probably because old fashioned typewriter keyboards contain = not ← .

Let's change instruction 22Ø PRINT X; by changing the semicolon to a comma. To do this, we simply type

    22Ø PRINT X, [ENTER]

If you now type

    LIST [ENTER]

the new program will be displayed below the LIST instruction. Please do so before continuing. Then type RUN [ENTER] .

9

Note that now our program prints out its values in four nice columns.

Isn't it impressive?

You may wonder what would happen if you did not have either the comma or the semicolon after PRINT X. If so, experiment a bit and find out.

Depress |BREAK| to get the computer's attention.

Type   LIST |ENTER|

Type 22∅ PRINT X |ENTER|

Type   LIST |ENTER|

Type   RUN |ENTER|

Depress | BREAK | to get the computer's attention.

Do some other experiments on your own. For example, change 23∅ X=X+1 to 23∅ X=X+.5 or 23∅ X=X+2 and see what happens when you type

   RUN |ENTER|.

Remember, about the only thing you might do that would hurt the TRS-80 would be to drop part of it or spill a beverage over it. Otherwise, it is pretty rugged, as long as you don't open the case.

KEEP YOUR COKES AWAY! SPILLS ARE HAZARDOUS TO MY HEALTH.

A Word of Warning:

    It is always well to check that you have typed what you think you have typed before depressing the |ENTER| key... In most cases it is easy to correct an error by simply retyping that statement number and instruction, but if you should happen to have typed LLIST |ENTER| instead of LIST |ENTER| you may have a problem you will need to call your instructor to fix. LLIST is also a perfectly valid instruction— it tells the TRS-80 to list your program on the attached line printer— but if your TRS-80 has no line printer attached (and turned on), the computer will not accept another instruction until it executes the impossible instruction LLIST |ENTER| . To all appearances your computer will "lock-up." Your instructor can get you out of this difficulty without having to turn off the computer. If you are without an instructor, open the small door on the back left top side of the keyboard, and press the button inside.

If your TRS-80 has "keyboard bounce" (printing multiple letters on one keystroke), it means the contacts are dirty and no one inserted the "keyboard fix" program when the TRS-80 was turned on. Ask for help if you have excessive troubles with keyboard bounce. Each TRS-80 has a keyboard-fix tape and manual packed with it. If you do not have an instructor, consult the manual that accompanies the keyboard fix tape.

H A V E   F U N ! ! !

SUMMARY OF LESSON 1

Let's see what you have learned thus far...

Depress BREAK key to get computer's attention if it is busy.

Type NEW   ENTER   to clear out everything and start over.

Number each instruction and put a space between the number and its instruction.

(If you fail to number an instruction the TRS-80 will perform that instruction right then, but not store it for later use. Try typing PRINT X, Y, T, L ENTER *.)*

You can erase errors by using the ⇐ key as a "backspace" if you have not depressed ENTER .

If you have depressed ENTER it's easy to change instructions, merely retype the number and new instruction. You may insert additional instructions between those already in use by using statement numbers that fall between those already used.

Holding down SHIFT while you depress ⇐ erases the entire line.

The symbol >_ indicates the computer is ready to receive typed instructions from you.

If the >_ symbol is not there, depress ENTER or BREAK   ENTER .

In the PRINT instruction, the use of a semicolon ; between or after variables will give a "packed format", while the use of a comma, produces four columns. No symbol gives one output per line (wasteful).

You may halt the output (temporarily) by depressing the ↑ if you have a Level I.   If you have a Level II, hold down SHIFT and depress @ . Depress any key except SHIFT or BREAK   to continue.

LIST   ENTER produces a listing of the program currently in storage.

Among your output you may discover some strange-looking numbers, such as 1.23456E+Ø8 or 2.22222E-Ø5. The computer displays only six significant decimal digits, and shows the magnitude by appending an E+nn or E-nn to indicate the power of ten by which the preceding fraction should be multiplied.

Thus

$1.23456E+\text{Ø}8 = 1.23456*10^{+\text{Ø}8} = 123456\text{ØØØ}.$ *(But recall this may be in error by ±5 in the seventh place, that is, by ±5ØØ.)*

$2.22222E-\text{Ø}5 = 2.22222*10^{-5} = .\text{ØØØØ}222222$ *(±.ØØØØØØØØØØ5)*
$3.45678E+11 = 345678\text{ØØØØØØ}$
$-4.68\text{Ø}24E-\text{Ø}7 = -.\text{ØØØØØØ}468\text{Ø}24$

This convenient notation, a cousin of the so-called "scientific notation," is used on most modern computing systems, including many hand-held calculators.

PRINT X,Y;Z;T will print values of X   Y Z T , spaced as shown.

CLEAR or C L S ENTER will clear the screen.

SHIFT   ⟹ will produce double size letters on the screen (see
                                                      problem 13).

Usually CLEAR is depressed just before using SHIFT   ⟹ .

PRACTICE SESSION 1

Problems 1 to 6 make use of the program discussed in the text. Start by typing it in, LISTing the program and RUNning it to check that it is functioning properly before you start the problem set.

depress [BREAK] key, then type:

```
NEW    [ENTER]
210    X = 1    [ENTER]
220    PRINT X;    [ENTER]
230    X = X + 1    [ENTER]
240    GOTO  220    [ENTER]
```

LIST [ENTER]

RUN   [ENTER]

[BREAK]

CONT [ENTER]

[BREAK]
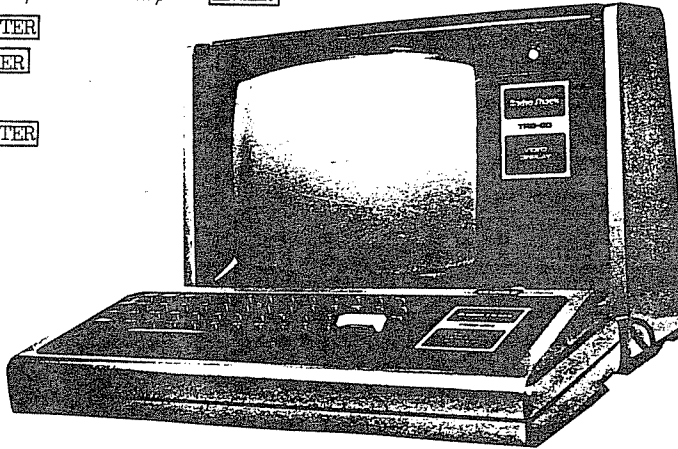
Now you are ready to start the practice set.

1. Type in the program (using 220 PRINT X;) first given in this lesson. Run it a few times with 230 X=X+1 replaced by each of the following in turn.

```
        230   X=X+.5    [ENTER]
  or    230   X=X+ 5    [ENTER]
  or    230   X=X+ 2    [ENTER]
  or    230   X=X+.1    [ENTER]

Type  LIST   [ENTER]
      RUN    [ENTER]
```

each time you change instruction 230. Note any peculiar behavior and think about it a bit. Discuss it with your instructor or a fellow computer buff.

13

2. In the program above, replace 21∅ X=1 with 21∅ X=7 and rerun the program.

3. Forecast the output of the program.

   ```
   21∅  X=1
   22∅  PRINT X,
   23∅  X=X+X
   24∅  GOTO 22∅
   ```

   Try it out and test your forecast. What about column spacing?
   The error message ?OV ERROR IN 23∅ indicates that the arithmetic
   operation in instruction 23∅ produced a result too large for the
   ~~computer to handle namely larger than 10³⁸~~ —which is a number
   larger than the total number of atoms in the solar system.

4. What output would you expect the program of problem 3 to produce if
   24∅ GOTO 22∅ were replaced with 24∅ GOTO 21∅?

   Try it out and verify your forecast, or explain why the computer did
   not do as you forecast.

   What would happen if you used 24∅ GOTO 23∅?
   Be sure you understand these points.

5. Let's try another program. Depress [BREAK] if needed. Then type
   NEW [ENTER].
   If you know that most computing systems use * to indicate multipli-
   cation, you should be able to forecast what the following program
   will produce.

   ```
   21∅  X=2
   22∅  PRINT X,
   23∅  X=X*X
   24∅  GOTO 22∅
   ```

   Make a forecast. Then run the program. If the results are unexpect-
   ed, reread problem 3.

6. Replace instruction 22∅ PRINT X, in problem 5 with 22∅ PRINT X, X+5
   and see what happens. Before running the program, try to forecast
   the output of the first 5 lines.

7. Try some experiments on your own. Don't be afraid--you are not apt
   to harm the TRS-80 computer providing you don't eat or drink in the
   computer room and don't move or bump it while it is connected. Be
   sure to depress [BREAK] and type NEW [ENTER] before you start a new
   program.

   > *The new instructions used in problems 8,9,10 will be discussed
   > in detail later. Now all you need to know is that
   > IF RND(∅)<.∅5 THEN CLS is pronounced "If 'random number'
   > is less than .∅5 then clear the screen."*

14

8. Let's try another program. Depress BREAK if needed. Then type
NEW ENTER.

Level II BASIC

1Ø PRINT @ RND(1ØØØ), "HI *type your name*"; ENTER
15 IF RND(Ø)<.Ø5 THEN CLS ENTER
2Ø GOTO 1Ø ENTER

RUN ENTER

Level 1 BASIC

*If you have a
Level 1 BASIC re-
place* @ *with*
A T *and it
will behave
similarly.*

9. Experiment a bit by changing instruction 15 IF RND(Ø) < .Ø5 THEN
CLS to one of those suggested below.

    15 IF RND(Ø) <.Ø1 THEN CLS
or 15 IF RND(Ø) <.1 THEN CLS   [*Don't forget to depress* ENTER
or 15 IF RND(Ø) <.5 THEN CLS   *each time you change an in-*
or 15 CLS   *struction.*]

10. Try combining the program in the text with that of problem 8, with
instruction 2Ø omitted, giving:
   1Ø PRINT @ RND(1ØØØ), "HI *type your name*";
   15 IF RND(Ø) < .Ø5 THEN CLS (*Type* 2Ø ENTER *to delete instr.* 2Ø.)
   21Ø X=1
   22Ø PRINT X;
   23Ø X=X+1
   24Ø GOTO 22Ø

11. Change the instruction 24Ø GOTO 22Ø to 24Ø GOTO 1Ø in problem
10. Forecast the output before you RUN the program. If your fore-
cast was not valid, try to find a why before reading problem 12.

12. Delete instruction 21Ø X=1 by typing 21Ø ENTER. When >_
appears, insert 5 X=1. Type LIST ENTER to check your new pro-
gram. Before you RUN the program, try to forecast the result. Why
is it different than that of problem 11 which used the same in-
structions, but placed differently in the program?

13. If you have a Level II BASIC, please try the following sometime
when you have a program that does not contain CLS in the computer.

Depress BREAK if needed.
Depress CLEAR
Hold down SHIFT and depress ⟹
Type LIST ENTER
Notice that your program is listed in characters twice as large as
before.
Now type RUN ENTER
The output is also double size. This is handy if you are using the
computer with a class or group. Double size will last until CLEAR

15

key is depressed or until you execute a CLS instrction. To get the
regular size back, depress [BREAK] and then [CLEAR].
If you fail to clear the screen before you depress [SHIFT] [==>],
your double-size letters and numbers obliterate half of the material
on the screen to make room for the larger letters, but any material
printed after that will be complete.

14. Have fun. Make your own variations.
    Forecast the result before you run each program.
    Try NEW [ENTER]

        210 X = 5 [ENTER]

        220 PRINT X; X+5; X+10; X+15; [ENTER]

        230 X = X+20 [ENTER]

        240 GOTO 220 [ENTER]

    Now investigate what happens if you omit the final ; in line 220.
    What happens if you change the semicolons to commas in line 220
    (with and without the final comma)?

15. Can you write a program that will start with X = 1 and count by
    threes, with exactly four numbers on each line? There are several
    correct ways to write such a program, so when you are finished,
    compare your program with that of a friend who also has a computer.

16. Try the following program.
        NEW [ENTER]

        100 X = 2 [ENTER]

        110 PRINT X, X*X, X*X*X, X*X*X*X   [ENTER]

        120 X = X+1 [ENTER]

        130 GOTO 110 [ENTER]

        RUN [ENTER]

16
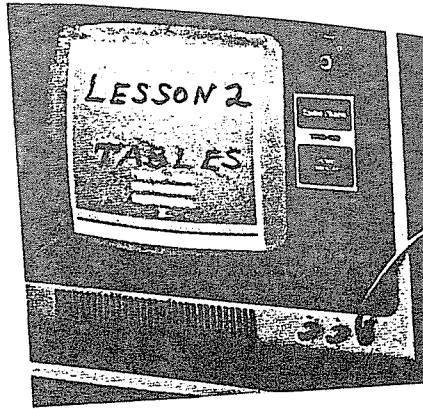
In our first lesson, we learned to make the computer
count, using various step sizes
to print in different formats, depending upon whether we used
      PRINT X
 or   PRINT X;
 or   PRINT X,

We also learned the essential handling of the computer using BREAK,
ENTER and ◄═══ "backspace" keys
and to type NEW ENTER   to clear out old program and data, ready to
accept new ones.

We rewrite the counting program using a slightly different philosophy
below (for reasons you will soon appreciate.) We shall use B = begin-
ning value, S = stepsize by which X will be increased, and F = final
value, after which we wish the computer to stop.

```
NEW  ENTER
  2    B=1  ENTER
  4    S=1  ENTER
  6    F=1ØØ  ENTER
 1ØØ   X = B  ENTER
 11Ø   PRINT X;  ENTER
 12Ø   X = X+S  ENTER
 13Ø   IF X < F THEN 11Ø  ENTER
```

Instruction 13Ø examines the current value of X and if X is less than
F, sends the program back to instruction 11Ø. If you feel at all in-
secure about this program, put it on the computer and RUN it before
continuing.

The real advantage of this technique is that we can easily change
the step size by changing instruction 4 to   4 S = .5 ENTER

Indeed, we may rewrite the program slightly to input values of B,
S, and F from the keyboard instead of assigning them in instructions 2,
4 and 6.

We do so next, using the instruction   INPUT B,S,F . When the computer
executes this instruction it will display a question mark on the TV
screen and wait for you to type in three values, separated by commas.
The values will be assigned, in the order typed, to variables B,S, and F.
These letters were used as they are the initial letters of the phrases
Beginning value, Step size, and Final value.

The revised program is:

```
NEW   ENTER
 90   INPUT B,S,F  ENTER
100   X=B  ENTER
110   PRINT X;   ENTER
120   X=X+S  ENTER
130   IF X < F  THEN 110  ENTER
140   PRINT  ENTER
150   PRINT  "END OF TABLE WITH B,S,F=";B;S;F  ENTER
160   PRINT : PRINT  ENTER
170   GOTO 90  ENTER
RUN   ENTER
```

Again, note the use of instruction

130 IF X < F THEN 110.

As long as X is less than F, the program loops back to 110 PRINT X;
but when X ≥ F, the program continues with statements 140, 150, 160, and
170 which upspaces the printed matter, prints an END OF TABLE message,
and then sends the computer back to 90 for a new set of input values.

Instruction 150 PRINT "END OF TABLE WITH B,S,F =";B;S;F is particularly
interesting. It combines the printing of a message in quotes, "   "
with the printing of the values of the variables B,S and F, which must
not be included within the quotes.

Try this program on your TRS-80 before continuing. Begin by typing
RUN ENTER . When a ? appears on the screen, type 1, 1, 100 ENTER.
Watch the output. When a ? appears at the bottom of the screen, type
2, .5, 10 ENTER or some B, S, F values of your own choice.

Let's write a program to create a table of values of

X X^2 X^3 X^4

When our program is run, it will accept values B, S, F from the keyboard. It will set X=B and print out values of X, X^2, X^3, X^4 for that value of X. Then the instruction X=X+S will add S to the current value of X and print another set of X, X^2, X^3, X^4 for the new value before adding another S to the current X.

This loop will continue until X exceeds the value of F (X>F) in which case the program will PRINT a blank line (16∅) followed by (17∅) an END OF TABLE message.

One way to do it is:
NEW   [ENTER]

```
1∅∅   PRINT "PLEASE TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE "; [ENTER]
1∅5   INPUT B,S,F  [ENTER]
11∅   X=B  [ENTER]
12∅   Y=X*X  [ENTER]
13∅   PRINT X, Y, X*Y, Y*Y  [ENTER]
14∅   X=X+S  [ENTER]
15∅   IF X < = F THEN   12∅   [ENTER]
16∅   PRINT  [ENTER]
17∅   PRINT "END OF TABLE"  [ENTER]
```

RUN [ENTER]

Let us examine the program in detail.

The instruction

1∅5   INPUT B,S,F
      causes the computer to type a ? and wait for the user to type
      three values separated by commas and depress the [ENTER] key.

Instructions 1∅∅ and 1∅5 can be combined into a single statement, but we shall not do so yet.

Instructions

11∅   X=B
12∅   Y=X*X
13∅   PRINT X, Y, X*Y, Y*Y

do just what you learned to expect in lesson one. Note that by using commas between output, we obtain four neat columns. Also note that the use of Y=X*X saves half of the multiplications that would be required by
PRINT X, X*X, X*X*X, X*X*X*X   since Y*X = X^3 and Y*Y = X^4.

The instruction

14∅   X=X+S
      merely increases the current value of X by the amount in S.

19

Next we use the branch instruction

150 IF X<=F THEN 120
     This instruction compares the values stored in X and in F. If the
     value in X is less than or equal to (<=) the value in F, the pro-
     gram loops back to instruction 120. Otherwise the instruction which
     follows instruction 150 (namely 160 PRINT) is executed next.

160 PRINT
170 PRINT "END OF TABLE"

     Instruction 160 merely prints a blank line, while instruction 170
     prints the message indicated in quotes. If after entering the pro-
     gram, you depress [CLEAR] type LIST [ENTER] , the computer display
     will show

```
>   LIST
100  PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
105  INPUT B,S,F
110  X=B
120  Y=X*X
130  PRINT X, Y, X*Y, Y*Y
140  X=X+S
150  IF X<= F  THEN 120
160  PRINT
170  PRINT "END OF TABLE"
READY
>_
```

If you now type RUN [ENTER] , the computer will respond

> RUN
TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE?

If you respond with 1,1,5 [ENTER] , the table of X, X^2, X^3, X^4 gives the
below results:

```
1    1     1     1
2    4     8     16
3    9     27    81
4    16    64    256
5    25    125   625
END OF TABLE
READY
>_
```

If you type RUN [ENTER] again, the computer will again respond with

TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE?

and if you type 1,.5,8 [ENTER] , the screen will fill up and "scroll out
the top" displaying

20

```
⎛1.5        2.25         3.375          5.∅625
⎜2          4            8             16
⎜2.5        6.25         15.625        39.∅625
⎜3          9            27            81
⎜3.5       12.25         42.875        15∅.∅63
⎜4         16            64           256
⎜4.5       2∅.25         91.125        41∅.∅63
⎜5         25           125           625
⎜5.5       3∅.25        166.375        915.∅63
⎜6         36           216          1296
⎜6.5       42.25        274.625       1785.∅6
⎜7         49           343          24∅1
⎜7.5       56.25        421.875       3164.∅6
⎜8         64           512          4∅96
⎜
⎜END OF TABLE
⎜READY
⎝>_
```

So, you see our computer works with fraction values as well as with whole numbers.

Note that $(2.5)^4$ = 39.0625 which is correct, while for $(3.5)^4$ the computer prints 150.063 instead of the correct value 150.0625. This is because, although the computer actually contains more than six digits of accuracy internally, it only <u>displays</u> six digits of accuracy. This will prove to be both a blessing and a curse, as we shall see.

If you are using a TRS-80 with Level II BASIC, it is quite feasible to obtain additional accuracy on it by declaring X and Y as DOUBLE PRE-CISION variables, but to do so now would only distract our learning procedure. Problems in "limited precision" and "rounding" also occur in double precision arithmetic--although not as frequently.

Let us type another set of values into our table. First, of course, depress CLEAR and type RUN ENTER. When the computer prompts you, type ∅,.1,1 ENTER. You should obtain:

```
∅         ∅           ∅             ∅
.1        .∅1         1E-∅3         1 E-∅4
.2        .∅4         8E-∅3         1.6E-∅3
.3        .∅9         .∅27          8.1E-∅3
.4        .16         .∅64          .∅256
.5        .25         .125          .∅625
.6        .36         .216          .1296
.7        .49         .343          .24∅1
.8        .64         .512          .4∅96
.9        .81         .729          .6561
```

Let's modify our program slightly by adding another instruction,
1Ø6 CLS.
To do so

Depress [BREAK] and [ENTER] (if necessary) to

obtain ⌈ READY
      ⌊ >_   on your screen and then type

1Ø￠ CLS [ENTER]

Next, depress [CLEAR] and type LIST [ENTER] to view the current program,
which should start

```
1ØØ   PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
1Ø5   INPUT B,S,F
1Ø6   CLS
11Ø   X=B
...
...
...
```

with the new instruction 1Ø6 tucked properly between instruction 1Ø5 and
11Ø.
Isn't that neat?

    The instruction CLS is called <u>clear screen</u>. It clears the screen
for you whenever it is executed. Try several B, S, F values of your own
choice. Note that if you use B, S; F = Ø, .1, 5, the table scrolls off
the top before you can read it. You can stop this at any time you wish.

<u>In Level II BASIC</u>

Hold down [SHIFT] and depress [@]
to stop the program. To continue
the program, depress [@] without
shift (or most any other key).

<u>In Level I BASIC</u>

Depress and hold down the [↟] key
To continue the program, release
the [↟] key.

<u>If you prefer</u>, depress [BREAK] to stop the program. Type CONT [ENTER]
to continue.

It would be nice to have headings at the top of the column of X, X^2, X^3, X^4 we are printing. It is not hard. We add:

1Ø8 PRINT "X   SQUARE   CUBE   4-TH POWER"   ENTER

So now our program reads:

```
1ØØ  PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
1Ø5  INPUT B,S,F
1Ø6  CLS
1Ø8  PRINT "X            SQUARE        CUBE       4-TH POWER"
11Ø  X = B
12Ø  Y = X*X
13Ø  PRINT X, Y, X*Y, Y*Y
14Ø  X = X + S
15Ø  IF X<=F THEN 12Ø
16Ø  PRINT
17Ø  PRINT"   END OF TABLE"
```

Type LIST ENTER
to list your program and see if it agrees.

Then type RUN ENTER
and observe the output when you enter B, S, F = 2, 1, 9

It should be

X	SQUARE	CUBE	4-TH POWER
2	4	8	16
3	9	27	81
.	.	.	.
.	.	.	.
7	49	343	2481
8	64	512	4Ø96
9	81	729	6561

If your headings do not seem to be lining up properly, change the spacing inside the quotation marks in the instruction

1Ø8 PRINT "X   SQUARE   CUBE   4-TH POWER"

until it pleases you.

Note that the instruction

15Ø IF X <= F THEN 12Ø

makes a <u>decision</u> by comparing the values of X and F. (IF X <= F THEN 12Ø) The <u>IF</u> instruction produces a branching in the path of the instructions, and takes the branch path (loop) instead of continuing on if the given condition is satisfied. This is indeed a powerful idea.

Try it again with B, S, F = 2, .5, 7.

·

2 3

As an experiment to test your understanding, let us insert another instruction.

125   Y = Y*Y   *(Note: The 1Ø8 |ENTER| deletes instruction 1Ø8, which*
1Ø8 |ENTER| *would have printed erroneous headings as previously*
              entered.)

You should forecast the output of the new program and insert new headings in instruction 1Ø8 before running the program.

```
1ØØ   PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
1Ø5   INPUT B,S,F
1Ø6   CLS
1Ø8         (Insert new headings here.)
11Ø   X = B
12Ø   Y = X*X
125   Y = Y*Y
13Ø   PRINT X,Y,X*Y,Y*Y
14Ø   X = X + S
15Ø   IF X< = F THEN 12Ø
16Ø   PRINT
17Ø   PRINT"  END OF TABLE"
```

The actual output for B,S,F = 2, .5, 1Ø is:

2	16	32	256
2.5	39.Ø625	97.6563	1525.88
3	81	243	6561
3.5	15Ø.Ø62	525.219	22518.7
4	256	1Ø24	65536
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.

As you experiment a bit, you will find the computer occasionally switching to an odd-looking number format, perhaps

$$1.23456E+\emptyset 8$$

This is called the "exponential" or "scientific" notation. The E+08 means to multiply 1.23456 by $1\emptyset^8$ (i.e., move the decimal point 8 places to the right).
Similarly 1.23456E-Ø4 would mean to multiply by $1\emptyset^{-4}$ (i.e., move decimal point 4 places to the left).

4.21653E+12 = 421653ØØØØØØ (with a possible error of ±5 in the 7th place)

7.65432E-Ø8 =.ØØØØØØØ765432 (with a possible error of ±5 in the 14th place)

If you are familiar with trigonometry and have Level II BASIC, you may wish to substitute.

1Ø8   PRINT "X", "SIN(X)", "COS(X)", "EXP(X)"

13Ø   PRINT X, SIN(X), COS(X), EXP(X)

There is no need to retype the entire program; just get

```
READY
>_
```

and type in

1Ø8   PRINT "X", "SIN(X)", "COS(X)", "EXP(X)"   [ENTER]
13Ø   PRINT X, SIN(X), COS(X), EXP(X)   [ENTER]

Run the revised program.

Note: we obtained the spacing desired in statement 1Ø8 by enclosing each column heading in quotation marks and separating the columns with commas, just as we did in statement 13Ø which prints the values. This is easier than using just one pair of quotation marks.

SUMMARY OF LESSON 2

What have you learned in Lesson 2?
Our new program extended your knowledge of the PRINT instruction to include the possibility of printing several different values from the same instruction, and of doing computation inside of the PRINT instruction itself.

You really do have a lot of computing power at your fingertips now.

KEYBOARD COMMANDS

| ENTER | Enter line from screen to computer. |

| CLEAR | Clear the screen |

| BREAK | TRS-80, stop what you are doing and pay attention. Type CONT ENTER to continue. |

| ⟸ | Backspace and erase. |

| SHIFT | ⟸ | Erase entire line. |

↑ (for Level I)
SHIFT @ (for Level II) STOP! *(Release the* ↑ *key, to continue.)* *(Depress any key, without using* SHIFT *to continue.)*

| NEW ENTER | Erase current program and data, clear screen and pay attention. |

| RUN ENTER | Run the program now in memory. |

| LIST ENTER | List the program now in memory. |

| SHIFT ⟹ | To get double-sized output ; CLEAR to return to single size. |

BASIC INSTRUCTIONS DISCUSSED THUS FAR

X = B

Y = A*X*X + 4.6*X − 3.172

} *Computes value on right and stores it in the location named at left of equals sign.*

PRINT X,Y;Z; T,X*Y

PRINT N; "VALUES OF X TOTAL"; Y

} *Prints values of variables or expressions indicated. Also prints any message given inside of quotation marks. Semicolon is for narrow spacing, comma for wide spacing.*

INPUT "TYPE X VALUE"; X

Types message in quotes, followed by a question mark and then accepts values from keyboard and stores them in location indicated.

IF X < = F THEN 12Ø

Compares values of variables X and F. If X value is less than or equal to F value, then instruction 12Ø is executed next. Otherwise, the instruction following the IF instruction is executed next.

+   add
−   subtract
*   multiply
/   divide

PRACTICE SESSION 2

     As in Lesson 1, you should begin by typing in, listing, checking and running the main program under discussion in this Lesson, namely:

```
100   PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
105   INPUT B,S,F
106   CLS
108   PRINT " X              SQUARE        CUBE        4-TH POWER"
110   X = B
120   Y = X*X
130   PRINT X,Y,  X*Y, Y*Y
140   X = X + S
150   IF X <= F THEN 120
160   PRINT
170   PRINT "      END OF TABLE"
```

1a. Run the textbook program given above.

1b. Change instruction 140 X=X+S
               to   140 $\overline{X} = \overline{X} + S$
     and run the program. *(Do you need to also change instruction 108?)*

2.   What happens if the program in the text is given B,S,F as 0, 2, 7 so that the step misses F = 7? Does it stop at 6 or at 8?

3.   a.   Write a program of your own to create a table of values of

$$X \qquad X+3 \qquad (X+3)^2$$

        for $X = 1, 2, 3, \ldots, 10$

    b.   Modify your program so it will accept input values B,S,F and print out
$$X \quad ,X+3 \quad , \quad (X+3)^2$$

        for X values from B to F in steps of S.

4.   Write some programs of your own choice using the instructions we have discussed.

5.   Write a program to INPUT the radius R, of a sphere, and print out on one line the radius, volume and surface area of the sphere using

$$\text{Volume} = \frac{4}{3}\,\pi R^3 \qquad\qquad \text{Surface Area} = 4\pi R^2$$

$$(\textit{Where } \pi = 3.14159\ldots)$$

6. Modify your program of problem 5 to INPUT values: B, S, F
   (for Beginning value, Step size and Final value) and PRINT a
   table of values of the radius, volume and surface area of spheres
   of radius B, B+S, B+2S, B+3S,...,F.

7. Write a program to convert Fahrenheit temperature to Celsius.

$$C = \frac{5}{9} (F - 32)$$

8. Write a program to convert temperature in degrees Celsius to
   Fahrenheit temperature.

$$F = \frac{9}{5} (C) + 32$$

9. Rewrite instruction 1Ø8 in the program that produces powers of
   numbers, so that the headings are valid after instruction 125 Y = Y*Y
   is also inserted. Check it by using B, S, F = 2, 1, 4

10. If you have LEVEL II BASIC, try adding the following statement to the
    program that makes tables of x, x^2, x^3, x^4

        5 DEFDBL X, Y
    and RUN the program with B, S, F = 2, .5, 6

    Note that we no longer have problems with 4.5^4 = 410.0625 being
    rounded to 41Ø.Ø63 since we are no longer restricted to 6 decimal
    digits in our arithmetic. The   DEFDBL X,Y   instruction permits
    the TRS-80 to compute with and display up to <u>sixteen decimal digits
    of accuracy</u> when needed. We shall discuss this further when the
    additional accuracy is really needed. (What happens to the four
    columns of output?)

11. Write a program to create tables of values of
        x, x^3, x^6, x^9
    with proper headings for various values of B, S, F typed in by the
    user.

12. What output occurs in the program to produce a table of x, x^2, x^3, x^4
    if some or all of B, S, F are negative? Try B, S, F = 2, -.5, -3 and
    B, S, F = -2, 1, 5 and B, S, F = -1, -1, -6.

13. What happens when some of B, S, F are negative in the table of powers
    program that includes   125 Y = Y*Y ?

29

14a. If your BASIC includes a square root function   SQR( )   try
     using
     12Ø Y = SQR(X)
     13Ø PRINT X,Y,SQR(Y),SQR(SQR(Y))
     in the text program.

  b. If you don't have a SQR(   ) function, try
     12Ø Y = X*X*X
     13Ø PRINT X,Y,X*Y,Y*Y
     Be sure to alter instruction 1Ø8 to print a correct heading for the
     current table columns in either (a) or (b).

15. What would happen to the program if the instruction that prints the
    heading were changed to
    123 PRINT"X   SQUARE   CUBE   4-TH POWER"
    Try it out and see. After you understand what happened, and why,
    delete instruction 123 by typing

    123 [ENTER]

Notice: If mathematics frightens you, you can laugh at your fears now.
         With a computer to do the arithmetic, all you need to do is
         type in the formula and the computer will do the arithmetic for
         you. It really is easy to do problems that used to stump you.
         Try it ; you'll like it!

16. If you like mathematics, you may wish to start by proving that the
    area of a rhombus (a "stepped-on" square) is given by $\frac{1}{2}CD$ where
    C and D are the lengths of its diagonals. If you don't like math-
    ematics, just believe us that the area of a rhombus is half the
    product of its diagonals. (OK?)
        Write a program that will accept input values of C and D and
    print out the message
        THE AREA OF A RHOMBUS HAVING DIAGONALS ___ AND ___ IS _____.
    with the blanks filled in with the proper values. After your
    program is running properly, put in some extra instructions that
    will print out the message
        THE VALUES OF C AND D MUST BOTH BE POSITIVE.
    if anyone uses negative or zero values as input.
    Hint:
      1ØØ INPUT "PLEASE TYPE VALUES OF DIAGONALS C,D="; C,D
      11Ø IF C < 0 GOTO 5ØØ
      12Ø IF D < 0 GOTO 5ØØ
      13Ø A = .5*C*D

      15Ø PRINT "THE AREA OF A RHOMBUS HAVING DIAGONALS ";C;"AND";
           D;"IS";A
      16Ø PRINT
      17Ø GOTO 1ØØ

                    (continued on next page)

30

```
5ØØ   PRINT "THE VALUES OF C AND D MUST BOTH BE POSITIVE."
51Ø   PRINT
52Ø   PRINT
53Ø   GOTO 1ØØ
```

17. If N is the <u>number</u> of <u>sides</u> in a regular polygon and L is the <u>length</u>
    of each side, then the radius of the <u>inscribed</u> circle is

    $\dfrac{L}{2} \dfrac{\cos K}{\sin K}$ where $K = \dfrac{\pi}{N}$

    The radius of the <u>circumscribed</u> circle is

    $\dfrac{L}{2 \sin K}$ where $K = \dfrac{\pi}{N}$

    The area of any circle is given by πr^2 where r is its radius and
    $\pi = 3.14159...$ You should be able to create a program that would
    list

    Number of Sides / Area of Inscribed Circle / Area of Circumscribed
    Circle
    for polygons having 3 or more sides each of length 1 unit. Do so.
    Examine the output and explain what is happening. If you don't know
    any trigonometry, do not let it disturb you. You can still write
    this program. Use

    · K = 3.14159/N
    I = .5*COS(K)/SIN(K)
    C = .5/SIN(K)
        where I stands for the area of the inscribed circle and
        C stands for the area of the circumscribed circle.

18. A tetrahedron (triangular pyramid) has four faces each of which is an
    equilateral triangle whose sides are each of length L.

    The surface area = $1.73205L^2$

    The volume = $0.11785L^3$

    At L = 1, the surface area is larger than the volume.
    At L = 20, the surface area is smaller than the volume.
    Find the value of L that makes the number of square units in the
    surface as nearly as possible equal to the number of cubic units in
    the volume.

19. For what radius does a sphere have the same surface area (in square
    units) as volume (in cubic units)?

    $Vol = \dfrac{4\pi}{3} R^3$ Surface $= 4\pi R^2$

20. If (X1,Y1) and (X2,Y2) are the coordinates of two points, the dis-
    tance between the two points is given by the formula

    $D = \sqrt{(X1-X2)^2 + (Y1-Y2)^2}$ Write a program to accept the coor-
    dinates of two points and print out the distance between the points.
    Does the formula work if some of the values are negative?   .

31

22. CHALLENGE PROBLEM: A job pays you $1 for the first 8-hour day's work, $2 for the second day's work, $4 for the next day's work, $8 for the next day's work, etc., doubling your day's wages each day, if you continue to work without being tardy or absent. It starts over at $1 any day you are late or missing.

Find how many days you will have to work, without a tardy or absence to earn a total of a million dollars. How much would you earn the next day, if you were still on time?

You may write your own program if you prefer, or you may use the program below:

```
1∅   PRINT "DAY#", "PAY ON THAT DAY","TOTAL PAY THUS FAR"
1∅∅  S = ∅
11∅  D = 1
12∅  P = 1
13∅  S = S+P
14∅  PRINT D,P,S
15∅  D = D+1
16∅  P = P+P
17∅  GOTO 13∅
```

After you run your program, modify it so that you can INPUT a value of T for the number of minutes tardy in step 17∅ and then send the program to 13∅ if T = 0 but to 12∅ otherwise. Use T = 480 if absent.

```
17∅   INPUT T
18∅   IF T = ∅   THEN 13∅   ELSE 12∅
```

What effect does this have?

23. CHALLENGE PROBLEM: It is easy to observe that the sum of the first four natural numbers is 1+2+3+4 = 10 or that the sum of the first seven integers is 1+2+3+4+5+6+7 = 28. How many successive natural numbers beginning with 1 would you have to add together to reach or just exceed 1000?

24. CHALLENGE PROBLEM: Bob drops a superball from a 4th floor window 42 feet above the parking lot below. If on each bounce the ball ascends to half the height it fell on that bounce, how far will the ball travel (both up and down) when it hits the ground for the 10th time? The 50th time? The 100th time?
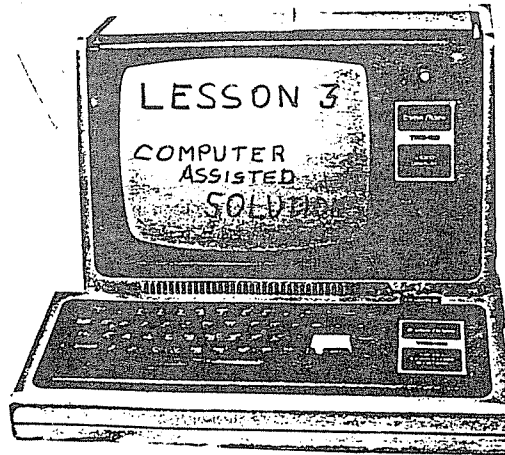
All right—it is now time to learn to do something you can't find in a set of tables and can't do without a computer—at least not conveniently.

Now, let's introduce a new instruction by reprogramming a "Table Maker" of Lesson 2.

A very useful instruction is the FOR...NEXT instruction. Try the following program on your computer.

```
100   FOR K = 3 TO 10
110      S = K*K
120         PRINT K,S,K*S,S*S
130   NEXT K
140   PRINT "END OF TABLE"
```

The FOR...NEXT instruction sets K = 3 and executes whatever instructions lie between FOR and NEXT; then K is advanced to K = 4 and the instructions between FOR and NEXT are again executed; then similarly for K = 5,6,7,8,9,10. When K = 10, the instructions between FOR and NEXT are again executed with K = 10. After reaching 130 NEXT K with K = 10, the program continues to instruction 140.

After you have run the above program, modify it to

```
80    PRINT "TYPE BEGINNING VALUE, END VALUE";
90    INPUT B,E
100   FOR K = B TO E
110      S = K*K
120         PRINT K,S,K*S,S*S
130   NEXT K
140   PRINT "END OF TABLE"
150   PRINT
160   GOTO 80
```

RUN the program for several different values of B and E. Now we are ready to write a more sophisticated program.

Let's write a program to help us <u>solve messy equations</u>.

Notice that no one said the program would solve equations. It won't. But it will do all the hard work and messy arithmetic for you. All you'll need to do is to tell it where to hunt for the roots, and it will do the hunting. Does that sound fair? Let's try it. If mathematics bores you, just hang in there for now. We'll be doing computer assisted art in Lesson 4 but for now I want you to realize how easy it really is to do mathematics with the help of a computer. If you can do 6th grade arithmetic, you <u>can do</u> this. Keep on reading, please.

It may not have occurred to you that computer programs have to be "built." They do not spring full grown from the mind, but develop slowly--and are modified as they develop--until a final program you would be willing to share with a colleague eventually results. One of the primary rules for good programs is

> Make your program <u>run</u> first,
>
> then make it fancy.

This is good advice. Let's use it.

Our problem is to write a program to help solve the equation
$5X^3 + 3X^2 - 2X - 5 = 0$.

We shall first write a program that accepts a value of X as input, and displays the value of X along with the corresponding values of
$Y = 5X^3 + 3X^2 - 2X - 5$.
To solve the equation, we must make Y = 0 (or as close as we can get).

The following preliminary program seems to provide the required evaluation.

```
210  PRINT "TYPE X-VALUE PLEASE";  ENTER
215  INPUT X   ENTER
240  Y = 5*X*X*X + 3*X*X - 2*X - 5   ENTER
250  PRINT X,Y  ENTER
280  PRINT  ENTER
290  GOTO  210  ENTER
```

Don't forget to type LIST ENTER to check your program.

Then type RUN ENTER
The program will print

    TYPE X-VALUE PLEASE ?

If you type   Ø ENTER

It will respond
    Ø -5

TYPE X-VALUE PLEASE?

If you type   1   [ENTER]

It will respond

   1   1

TYPE X-VALUE PLEASE?

The change of sign in Y from -5 to +1 shows, since Y is a continuous function, that there is a root somewhere between X = $\emptyset$ and X = 1 — possibly nearer X = 1.

If we were to graph $Y = 5x^3 + 3x^2 - 2x - 5$ we would know that for

X	$\emptyset$	1
Y	-5	1
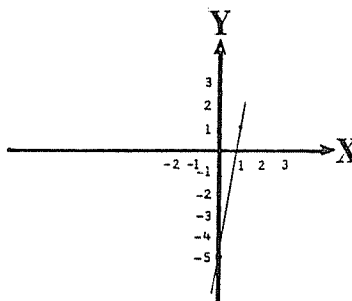
Our observation suggests that for some X-value between X = $\emptyset$ and X = 1, the value of Y must be zero. It is this X-value that corresponds to Y = $\emptyset$ that we seek.

Let's try $\emptyset$.8   [ENTER]
The computer responds

   .8   -2.12

TYPE X-VALUE PLEASE?

So we now know there is a root between
  X = $\emptyset$.8 and X = 1   (Why?)

so we type $\emptyset$.9   [ENTER]

and obtain

   .9   -.725

so the sign change (and root) comes
  between X = .9 and X = 1

We can now enter   .95   [ENTER]
and if the corresponding Y is positive, follow this with .94 etc. Between .944 and .945 we eventually obtain the root accurate to 5 or 6 significant digits. Try a few values yourself.
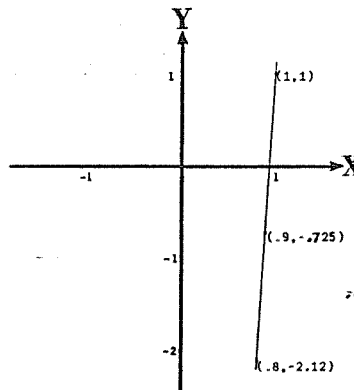
We can devise an even better helper program to solve equations. Let's modify our program so it accepts two values B and S (for Beginning value and Step size) and then prints out 11 pairs of values of X and Y before looping back to get another pair of starting values.

If you aren't sure why this will be a convenient change, just wait until you try the program given below.

We shall use our new FOR.... NEXT instruction. It is really a pair of instructions

```
FOR  K=1 TO 11
     . . . .
     . . . .
NEXT K
```

The instruction FOR K=1 TO 11 sets K = 1 and performs the instructions between FOR and NEXT (which may or may not involve K).

The NEXT K instruction then adds 1 to K (making K=2) and again performs the instructions between FOR and NEXT. Then 1 is again added to K, producing K=3, and the instructions between FOR and NEXT are executed again, and again, and again until finally K=11. The instructions between FOR and NEXT are performed once more, but this time whatever instruction directly <u>follows</u> the NEXT is executed.

The wee program

```
1ØØ   FOR K=1 TO 2Ø
11Ø      PRINT K,K*K
12Ø   NEXT K
```

will produce a table of integral values of K and K^2 for K between 1 and 2Ø inclusive.

The following program produces rather different results in Level I and Level II BASIC.
Try it on your computer.

```
1ØØ   FOR K=Ø TO 2Ø STEP .5
11Ø      PRINT K,K*K
12Ø   NEXT K
```

On Level I BASIC, it produces a table whose entries are Øs. This is because in Level I BASIC, STEP size must be an integer and STEP .5 is truncated to STEP Ø.

Level II produces a table of K, K^2 for K between Ø and 2Ø with values changing by .5.

Try it if you are unsure. The best way to learn about your computer is to compute.

Let us return to our first version of

```
21Ø   PRINT "TYPE X-VALUE PLEASE";
215   INPUT X
24Ø   Y = 5*X*X*X + 3*X*X - 2*X - 5
25Ø   PRINT X,Y
28Ø   PRINT
29Ø   GOTO 21Ø
```

Let us combine some of the tricks we learned in Lesson 2 to produce a table of eleven values of X and Y = 5*X*X*X + 3*X*X - 2*X - 5 starting with X = B and increasing the X value by S on each line.

Second try:

```
21Ø   PRINT "PLEASE TYPE BEGINNING VALUE, STEP SIZE";
215   INPUT B,S
22Ø   X=B
23Ø   FOR K=1 TO 11
24Ø      Y = 5*X*X*X + 3*X*X - 2*X - 5
25Ø      PRINT X,Y
26Ø      X=X+S
27Ø   NEXT K
28Ø   PRINT
29Ø   GOTO  21Ø
```

LIST ENTER and check your program.

Then type   RUN ENTER

The program will print

    PLEASE TYPE BEGINNING VALUE, STEP SIZE?

and you type   Ø, 1 ENTER

The output will be: Ø   -5 ──── (Note sign change!)
                           1   1 ←
                           2   43
                           3   151
                           4   355
                           5   685
                           6   1171
                           7   1843
                           8   2731
                           9   3865
                          1Ø   5275

Since there is a sign change between X = Ø and X = 1, you next type
Ø, .1 [ENTER] which produces

Ø	-5
.1	-5.165
.2	-5.24
.3	-5.195
.4	-5
.5	-4.625
.6	-4.Ø4
.7	-3.215
.8	-2.12
.9	- .724999
1	1 ←——— (Note sign change!)

A sign change between .9 and 1.Ø indicates a root there,

so you type .9, .Ø1 [ENTER] .

Then later you type .94, .ØØ1 [ENTER] which shows a root between
.944 and .945.

.94	- .Ø76280́6
.941	- .Ø59370́5
.942	- .Ø424247
.943	- .Ø254455
.944	-8.43143E-Ø3
.945	8.61549E-Ø3 ←——— (Note sign change!)
.946	.Ø256987
.947	.Ø428152
.948	.Ø599666
.949	.Ø771518
.95	.Ø943718

So now we know that the equation $5x^3+3x^2-2x-5=0$ has a root between
x = .944 and x = .945. You can easily obtain 5 or 6 places of accuracy,
if needed, but 3-place accuracy is frequently enough.

   To solve $7x^4 + 4x^3 + 2x^2 - 9x - 1ØØ$, you need only to change line
24Ø to

  24Ø Y = 7*X*X*X*X + 4*X*X*X + 2*X*X - 9*X - 1ØØ

Starting values Ø, 1 (and also Ø, -1) will each be helpful in solving
this problem.

Computer experts will prefer to write line 24Ø as

24Ø   Y = (((7*X + 4)*X + 2)*X – 9)*X – 1ØØ

which saves both typing and, more important, computing time, since it uses only four multiplications instead of 10, each time Y is evaluated. If you have had a course in algebra, please multiply out the expression to see that the same results are obtained. You can solve many different equations by changing line 24Ø.

What about solving $x^3 - 4.9x^2 + 6.6x - 2.6 = 0$ by changing line 24Ø to

24Ø   Y = ((X – 4.9)*X + 6.6)*X – 2.6

Note that input B, S = Ø, 1 produces three changes in sign. Find all three roots accurate to 4 or 5 significant digits.

If you have Level II BASIC, you may wish to try $Y = e^x \cos x^2 - 2.6$

24Ø   Y = EXP(X)*COS(X*X) – 2.6

This program is really quite a valuable tool. It will help you to solve almost any <u>continuous</u> (*e.g., no factor in the denominator that might be zero for X inside the interval in which you are seeking a root*) function set equal to zero.

Polynomials come under this classification--even polynomials in sin(x), cos(x), e^x, as well as polynomials in X. So do many other functions, but <u>not</u> equations like

$$\frac{3}{2x - 1} = 0 \quad \text{since} \quad y = \frac{3}{2x - 1} \quad \text{is discontinuous at } x = 1/2$$
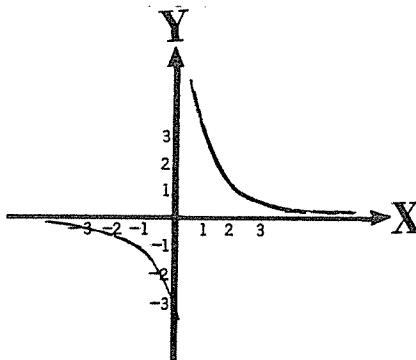
Remember: to change the equation being solved, you only need to change instruction 24Ø to

24Ø   Y = (whatever continuous function you are trying to make zero).

CORRECTING ERRORS discovered at RUN time.

You may have noticed that the TRS-80 will frequently detect errors in spelling or punctuation of instructions and will warn you of these errors by printing an ERROR message; consider the following erroneous program:

    1ØØ FOT K=1 TO 1Ø
    11Ø PRINT K,K*K
    12Ø NEXT K

in which FOR is misspelled FOT in line 1ØØ. If you type

    RUN ENTER

the computer responds

    ?SN ERROR IN 1ØØ
    READY
    1ØØ _

The first line tells you that there is an ERROR in statement 1ØØ. We shall talk more about ERROR types later on - for now you can usually spot the error by looking at statement 1ØØ.

For certain (but not all) errors, the computer will shift automatically into EDIT mode. This is indicated by displaying the line number where the ERROR has been detected just below the ERROR IN 1ØØ message.

If you depress the L key (without ENTER), the entire line will be displayed, and the line number repeated

    1ØØ FOT K=1 TO 1Ø
    1ØØ _

If you tap the space bar the line will appear character by character until the underline marks where the ERROR is:

    1ØØ FOT K=1 TO 1Ø
    1ØØ FO_

Now depress C (which stands for change); nothing visible will happen, but if you now depress the desired character R it will appear. If there are other errors, correct them too. Then depress ENTER

Type LIST ENTER and you will find the correction was made.

If you should need to delete a character depress D instead of C , the deleted character will appear between ! ! symbols. To insert a character depress I instead of C and then depress the desired

character. If you wish to change, delete or insert several characters
depress the number key before depressing the [C] , [D] , [I] key.
Thus [3] [I] will enable you to insert 3 characters. This is only
part of a set of powerful EDIT instructions available in Level II BASIC.
We shall discuss them in Lesson 8 , but you may peek now if you wish.

In many cases it is _much_ simpler to merely retype the line, rather
than using the EDIT instructions. In such a case if you type

RUN [ENTER] and get

?SN ERROR IN 1ØØ
READY
1ØØ _

Simply depress [ENTER]

This will display line 1ØØ, and display the >_ which you _must_ have
to enter an instruction. Depresssing [ENTER] will display

?SN ERROR IN 1ØØ
READY
1ØØ FOT K=1 TO 1Ø
>_

Then you merely retype the instruction

>1ØØ FOR K=1 TO 1Ø   [ENTER]

```
        The critical things to remember are:

The computer indicates it is ready to accept data by displaying    ?
Do not type anything but data if you have a   ?   displayed.

The computer indicates it is ready to accept an instruction by
displaying >_
Do not type an instruction unless you have a >_ displayed.

If in doubt depress   [BREAK] followed by   [ENTER]   to get

    READY
    >_
```

SUMMARY OF LESSON 3

We learned about the

   FOR K = 3 TO 1Ø

   .
   .
   .
   NEXT K

and

   FOR K = Ø TO 2Ø   STEP .5

   .
   .
   .
   NEXT K

instructions, and used them to develop a powerful program that can be used to help solve continuous equations.

We also learned how to correct errors that are detected by the computer at RUN time.

Perhaps the most important thing we learned in Lesson 3 is that mathematical problems are <u>not</u> inherently difficult. Even if mathematics troubled you in the <u>past</u>, with the able assistance of a microcomputer <u>YOU</u> <u>CAN</u> use plain old common sense to solve problems that would cause a college mathematics major difficulty if he tried to solve them without a computer.

PRACTICE SET 3

In problems 1 to 10, use the second equation solver program to help you find solutions of the problems given below.

1. $x^5 + 3x^4 + 2x^3 + 5x^2 + 3x - 25 = 0$

2. $x^5 - 31 = 0$

3. $7x^4 + 6x^3 + 10x^2 - 5x - 1 = 0$

4. $4x^3 - 8x^2 - 29x - 13 = 0$ *(Find three roots between -10 and +10.)*

5. $x^4 - 3x^2 - 4 = 0$

6. a. $x^4 - x^3 - 5x^2 - x - 6 = 0$

    b. $x^4 - x^3 - 5x^2 - x - 5 = 0$

7. $2x^4 + x^3 - 8x^2 - x + 6 = 0$

8. $6x^4 + 5x^3 - 14x^2 + x + 2 = 0$

9. $3x^3 + 4x^2 - 12x - 16 = 0$

10. $4x^4 - 13x^2 + 3 = 0$

11. a. $2x^4 - x^3 - 15x^2 + 6x + 17 = 0$

    b. $2x^5 - x^4 - 6x^3 + 3x^2 + 4x - 2 = 0$

12. Change the constant terms in some of the above equations and try again.

13. If you have a Level II BASIC, try some trigonometric equations like

$$X*SIN(X) - 2 = 0$$

$$X*EXP(X)*COS(X) + X*X - 4.5 = 0$$

The fascinating thing is you don't really need to understand trigonometry to solve the given (rather carefully selected) equations.

14. Forecast the output of the following program, then RUN it to check your forecast.

```
100   FOR  X=1 TO 10   STEP .2
110       PRINT X,X*X, X*X*X
120   NEXT X
```
Be sure to think about whether your computer has Level I BASIC or Level II BASIC.

43

15. Use the program

```
 9Ø   INPUT "PLEASE TYPE BEGINNING VALUE, END VALUE, STEP SIZE=";B,E,S
1ØØ   FOR X = B TO E     STEP S
1Ø5      Y=X*X
11Ø      PRINT X,Y,X*Y,Y*Y
12Ø   NEXT X
13Ø   PRINT
14Ø   GOTO 9Ø
```

16. The following program is supposed to accept a positive integer value
N and print out the values of N and N! = 1*2*3*...*N.
It seems to work the first time through, but then prints out non-
sense values for N! Try it and see. Then debug and repair the pro-
gram.

```
1ØØ   PRINT "N", "N-FACTORIAL"
11Ø   PRINT
12Ø   F=1
15Ø   INPUT "TYPE INTEGER WHOSE FACTORIAL YOU WISH" ; N
155   IF N<Ø  THEN 15Ø
16Ø   IF N=Ø  PRINT N, F          These test the input data...
17Ø   IF N<>INT(N)  THEN 15Ø      NOTE: INT(N) produces the integer
2ØØ   FOR K = 1 TO N              portion of N, so
21Ø      F = F*K                  17Ø  IF  N <> INT(N)  THEN 15Ø
22Ø   NEXT K                      looks at N and if N is not an
25Ø   PRINT N, F                  integer, the program goes back to
26Ø   GOTO 15Ø                    15Ø. Similarly, if N is negative,
                                  instruction 155  IF  N < Ø  THEN 15Ø
                                  sends the program back to instruc-
                                  tion 15Ø.
```

17. The following program is supposed to accept two integers N, D and
determine whether or not D is a factor of N. Does it work? If so,
why? If not, fix it. The heart of the program is lines 13Ø, 14Ø.

```
1ØØ   INPUT "TYPE IN TWO POSITIVE INTEGERS"; N, D
11Ø   IF N<D  THEN 5ØØ
12Ø   IF N=D  THEN 6ØØ
13Ø   Q=INT(N/D)
14Ø   IF D*Q <> N  THEN 4ØØ

2ØØ   REM  PROGRAM COMES HERE ONLY IF D IS A FACTOR OF N
21Ø   PRINT D;  "IS A FACTOR OF";N
22Ø   PRINT
23Ø   GOTO 1ØØ

4ØØ   REM     PROGRAM COMES HERE ONLY IF D IS NOT A FACTOR OF N
41Ø   PRINT D;  "IS NOT A FACTOR OF " ; N
42Ø   PRINT
43Ø   GOTO 1ØØ
```

17. (continued)

```
500   PRINT N;"=N IS SMALLER THAN D=";D;"HENCE D CAN'T BE A FACTOR OF N"
510   X=N
520   N=D
530   D=X
540   GOTO 130

600   PRINT "THE VALUES BOTH EQUAL"; N, "HENCE EACH DIVIDES THE OTHER"
610   PRINT
620   GOTO 100
```

18. In Lesson 2 you wrote programs to change temperature in Fahrenheit to temperature in Celsius and vice versa. Combine the ideas of your two programs into a single program that accepts B, S, E as Beginning value, Step size, and End value for T and then writes out two sets of tables side by side (four columns) using
   PRINT T;C,T;F   ←——————————(Note use of ; , ; in output here.)
   where T runs from B to E in steps of S and where
   C = (the Celsius temperature corresponding to T degrees in Fahrenheit)
   F = (the Fahrenheit temperature corresponding to T degrees Celsius)

19. Write a program to INPUT the hourly rate, R, and the number of hours, H, worked in a given week and print out the employee's gross wages for that week, assuming (s)he is paid time and a half for all time over forty hours in a given week.
   The output statement might be:
   PRINT "$";W;"FOR";H;"HOURS AT BASIC HOURLY RATE OF";R
   but you'll have to scratch your head a bit to compute the total for W, including the overtime payments, if any. Don't make the error of subtracting ten hours of "undertime" for an employee who only works 30 hours. It does not work that way.

20. Write $X^4 - 7X^3 + 5X^2 + 3X + 71$ as a series of nested parentheses.

21. Express $(((3*X - 7)*X +5)*X +2)*X + 13$ in a form without parentheses.

22. CHALLENGE PROBLEM: The sequence 1,1,2,3,5,8,13,21,...is created by writing 1,1 and then each succeeding term is the sum of the preceeding two terms, 1+1=2, 1+2=3, 2+3=5, etc.
   Write a program to compute the first 100 or so terms of this "Fibonnacci sequence."

23. CHALLENGE PROBLEM: Solve $X^2 - \cos X = 0$

24. CHALLENGE PROBLEM: $4x^8 - 2x^7 + x^6 - 3x^4 + x^2 - x + 1 = 0$
   presents an interesting puzzle for the mathematically-oriented. reader.

25. CHALLENGE PROBLEM: A dog is chasing a rabbit. The rabbit takes three jumps in the same length of time the dog takes two jumps, but each rabbit jump covers only half of the distance of a dog jump. The rabbit was 13 rabbit jumps ahead of the dog when the dog first spotted the rabbit and started after it. If both go in a straight line, how many more jumps will the rabbit take before the dog catches it?

26. CHALLENGE PROBLEM: How much must you invest now in an account paying 9% interest compounded quarterly to have $100,000 when you are 70 years old? (If you don't want to give your age away, figure it out for S. Marguerite who is 22 years old.)

27. CHALLENGE PROBLEM: Legend states that in 1626 Manhatten Island was purchased from the Indians for $24. If that $24 had been invested at 12% interest (a modest interest rate at that time) what would the $24 be worth today?
(How does this compare with the value of the land in Manhattan Island today?)

28. Here are two small "research" problems for you.

    CHALLENGE PROBLEM: The four-digit number 9801 has the unusual property that if you take the two-digit number formed by the first two digits, 98, and add it to the number represented by the last two digits, 01, then 98 + 01 = 99 and $(99)^2 = 9801$. Find all four-digit numbers that have this property.

29.a) CHALLENGE PROBLEM: A 4 by 4 rectangle (square) has the unusual property that its perimeter (distance around it) is the same as its area (both are 16). Your problem is to find another rectangle that also has the property that its length and width are both integers and its perimeter (2*(L+W)) is equal to its area (L*W).

   b) CHALLENGE PROBLEM: If you like mathematical thinking, see if you can prove that the two rectangles you found above are the only two such rectangles that exist.

30. CHALLENGE PROBLEM: The song "Twelve Days of Christmas" mentions various gifts that "my true love gave to me." Let's interpret the song so that "on the third day of Christmas, my true love gave to me three French hens, two turtle doves and a partridge in a pear tree." My true love gave me six presents (3 French hens, 2 turtle doves, and a partridge, if we don't count the pear tree as a gift). How many gifts in total did my true love give to me during the twelve days of Christmas? Write a program to determine the sum.

    Answer: 364, if you do not count the pear tree as a gift.

LESSON 4

ART & GRAPHICS

Computers are fun. They make great adult toys. As my daughter says, "The main difference between men and boys is the price of their toys." Well, a modern computer costs less than many adult toys — a Jaguar XKE, a sailboat, a motor boat, a golf cart or even a set of clubs can cost more than a computer. Of course, the computer is a very useful device (but so is an XKE) so let's just look at how to have fun with it.

If you are curious, you may have doped out how the little surprise program of Problem 8, Lesson 1, worked. Let's look at it.

(Remember to depress ENTER at the end of each line.)

Level II BASIC	Level I BASIC
NEW ENTER	NEW ENTER
1Ø PRINT @ RND(1ØØØ), "HI SUZANNE"	1Ø PRINT AT RND(1ØØØ),"HI SUZANNE"
15 IF RND(Ø) < .Ø5 THEN CLS	15 IF RND(Ø)<.Ø5 THEN CLS
2Ø GOTO 1Ø	2Ø GOTO 1Ø
RUN	RUN

Line 1Ø contains two new ideas. One of these is the idea of a random number generator. TRS-80 contains two types of random number generators:

One is called RND(Ø). RND(Ø) produces a random decimal number between Ø and 1.

The other is RND(n), where n is a positive integer. RND(n) produces a random integer between 1 and n inclusive.

To see how they work, use the programs on the following page.

```
1000 PRINT RND(0);
1010 GOTO 1000
RUN  1000  [ENTER] †
```

After you have a screen full of that, try

```
1000 PRINT RND(100);
1010 GOTO 1000
RUN 1000    [ENTER]
```

Later, change instruction 1000 to

1000 PRINT RND(10); *(Note: this still uses instruction 1010 GOTO 1000.)*

and type   RUN 1000   [ENTER]   again.

Each time you call RND(n) a random number is produced that is different from the random number produced last time, even though the same n is in the parentheses.

The second new idea in the instruction

10 PRINT @ RND(1000), "HI SUZANNE"

is the   PRINT AT __ , "_____" instruction.

For purposes of the   PRINT AT __ , "_____" the screen is divided into 1024 cells, sixteen rows of 64 elements each. Each cell exactly fits one letter plus the space between rows for that letter. The cells are numbered from 0 to 63 in the first row, 64 to 127 in the second row, etc. (See illustration on the following page.)

RND(0) TELLS ME TO GIVE YOU DIFFERENT RANDOM DECIMALS LIKE .739692, .109564, OR .512703

---
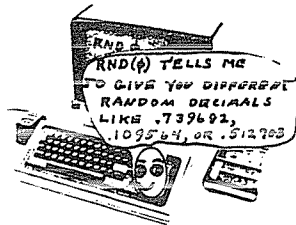
† *This uses   RUN 1000 [ENTER]   in place of   RUN [ENTER]   to make the computer skip any program in 10, 15, 20,...,999, and start RUNning at statement 1000. A neat trick.*

HI SUZANNE

Try the program

```
1000 INPUT N
1005 CLS
1010 PRINT  @  N, "* MARKS LOCATION";  N;
1020 GOTO 1000
RUN 1000
```

(Use AT *in place of* @ *on Level I BASIC.*)

The instruction

$$\text{PRINT} \begin{pmatrix} \text{AT} \\ \text{@} \end{pmatrix} n \qquad \qquad \text{use} \begin{cases} \text{AT } \text{in Level I BASIC.} \\ \text{@ } \text{in Level II BASIC.} \end{cases}$$

will start printing at location n and print whatever is called for. It prints any symbols it finds inside of quotes as symbols and the values of any variables it finds outside of quotes, providing the proper commas and/or semicolons are included.

For example, instruction 10 of the program in Problem 8 of Lesson 1

$$\text{10 PRINT} \begin{pmatrix} \text{@} \\ \text{AT} \end{pmatrix} \text{RND}(1000), \text{"HI SUZANNE"}$$

combines these two instructions. RND(1000) generates a random integer between 1 and 1000.

$$\text{PRINT} \begin{pmatrix} \text{@} \\ \text{AT} \end{pmatrix} \text{RND}(1000), \text{"HI SUZANNE"}$$

then prints whatever is in the quotes, starting at whatever random location RND(1000) generated. The next time, a different random integer is generated. The result is the statement in quotes is flashed all over the screen.

If you wish a snow storm, try

```
NEW   [ENTER]
1000 PRINT (@/AT)  RND(1020), "*";
1020 GOTO 1000
```

To keep the screen from scrolling use ; at the end of line 1000.

To occasionally wipe the screen clear, add

```
1010 IF RND(0)<.01 THEN CLS
```

The instruction RND(0) generates a random decimal number between 0 and 1. If the random number generated is less than .01 (which it will be about once in 100 numbers on the average), the "clear screen" instruction CLS is executed. Otherwise, CLS is ignored. In either case, the instruction 1020 GOTO 1000 is executed next.

Here are some interesting programs. Try them on your TRS-80.

```
NEW      [ENTER]
5 CLS
10 FOR L = 0 TO 960  STEP 64
15    FOR K = 1 TO 61
20       PRINT @  K + L, "HI your name"
25    NEXT K
30 NEXT L
40 GOTO 10
```

After you try that with several different names, change instruction 15 to

```
15 FOR K = 62 TO 5 STEP -1
```

and RUN it again.

Here is another FUN program. Please try it, then analyze what happens and why. The CHR$( ) instruction, discussed later in this lesson, displays the character corresponding to the value in the parenthesis.

```
NEW   [ENTER]
100 CLS
110 N=1
120 PRINT @ 970, " MOBILE ART FORM # "; N ;
130 FOR K = 129 TO 191
140    PRINT @  RND(959),CHR$(K) ;
150 NEXT K
160 N = N+1
170 FOR Q = 0 TO RND(N) : NEXT Q
180    IF N < 50 THEN 120 ELSE 100
```

Your TRS-80 has much finer graphic capabilities than we have been
using in PRINT $\left(\begin{array}{c} @ \\ AT \end{array}\right)$ _____ , "_____". Actually, each letter-
sized cell is broken up into six smaller rectangular cells, or *pixels*,
counting the space between lines, and each cell is individually address-
able. The system of numbering from $\emptyset$ to $1\emptyset23$ is too coarse for this use.
The individual spots are addressed by giving an X, and Y-coordinate with
$\emptyset \leq X \leq 127$ and $\emptyset \leq Y \leq 47$. The $(\emptyset,\emptyset)$ point is in the upper left edge
of the screen. X increases to the right. Y increases downward.
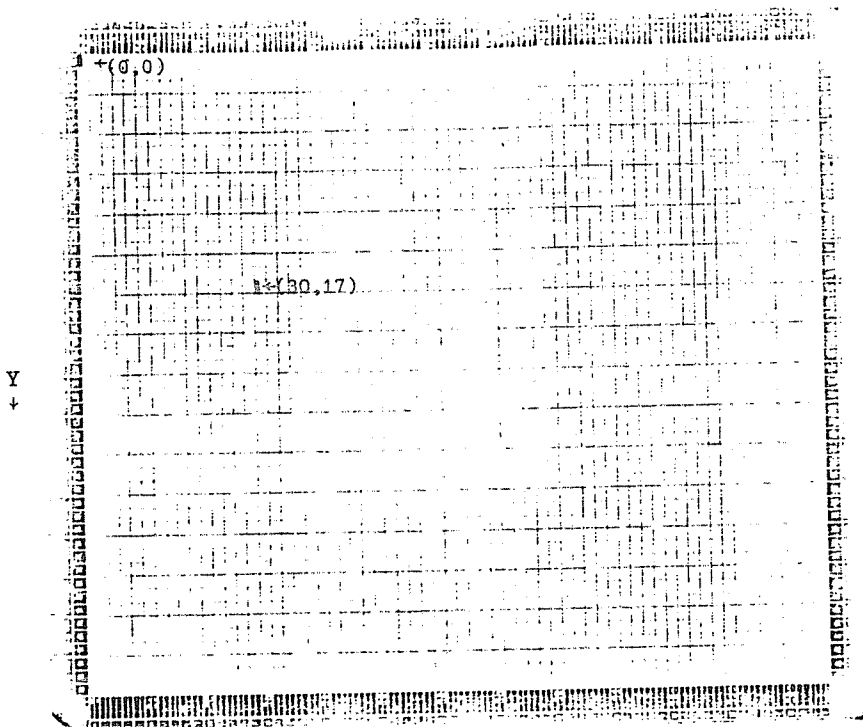
So, $(3\emptyset, 17)$ is 30 spaces to the right and 17 spaces down from $(\emptyset,\emptyset)$, as
shown above.

Try

```
1000 INPUT "X,Y=" ; X,Y
1010 SET(X,Y)
1020 GOTO 1000
RUN
```

There are three instructions used with these spots:

SET(X,Y)   This lights point (X,Y).

RESET (X,Y)   This darkens point (X,Y).

POINT(X,Y)   This is used to determine whether or not point(X,Y) is
             lighted. If lighted, it returns "logical yes", otherwise,
             "logical no", as used in an   IF....THEN   instruction.

```
100 CLS
110 FOR K = 0 TO 47
120     SET(K,K) : SET(K+60,K)
130 NEXT K
RUN
```

After you have RUN the above program, and understand why it produces
two wiggly diagonal lines, try adding

```
125 SET(120-K,K) : SET(60-K,K)
```

Before you run the new program

```
100 CLS
110 FOR K = 0 TO 47
120     SET(K,K) : SET(K+60,K)
125     SET(120-K,K) : SET(60-K,K)
130 NEXT K
```

see if you can forecast the pattern that will be produced. Then (and only
then) type RUN ENTER to check your forecast. This is one of the
best ways to learn about computers.

IF POINT(X,Y) THEN RESET(X,Y)   will test (X,Y) and turn it off, if it is
on. However, RESET(X,Y) also turns off point (X,Y) and takes less time.

Later, you will find uses for POINT(X,Y) in programs in which you will
need to test whether or not a given pixel is lighted.

    The following program will help you become familiar with the loca-
tion of the various (X,Y) rectangles (pixels). It lights a rectangle
chosen at random, and then permits you to input your estimate of the co-
ordinates (X,Y) of that rectangle. If you miss, a rectangle will blink
at the coordinates you chose to show you its location with respect to
the unblinking target. You may then input another guess.

```
100 R = 10 + RND(30)
110 C = 20 + RND(60)
120 CLS
130 SET(C,R)
140 INPUT "PLEASE TYPE YOUR ESTIMATE OF (X,Y) AS   X,Y ="; X,Y
150 IF(X=C)  AND (Y=R)  THEN 400

200 FOR K=1 TO 200
210    SET(X,Y)
220    RESET(X,Y)
230 NEXT K
240 GOTO 120

400 REM     HERE IF ESTIMATE IS CORRECT.
410 PRINT  @ 70,  "YOU ARE CORRECT.  CONGRATULATIONS."
420 FOR Q=1 TO 800  : NEXT Q
430 FOR Q=1 TO 50   : PRINT  @ RND(1000),  "YOU WIN"  : NEXT Q :  CLS
440 PRINT @  128,  "HERE IS ANOTHER POINT FOR YOU"
450 FOR Q=1 TO 500  :  NEXT Q
460 GOTO 100
```

It is a game...put it on your computer and play it. This is guaranteed
to improve your visualization of what is where on the pixel screen.

Here is another possibly useful technique.

Let us use our knowledge of graphics to create a bar graph or histo-
gram for the number of students attending The University of Oklahoma.

```
100 CLS
110 PRINT @ 6, "GRAPH OF ENROLLMENTS AT  THE UNIVERSITY OF OKLAHOMA"
120 REM    LABEL THE LINES FOR THE BAR GRAPHS
130 PRINT  @  128,   "1940"
140 PRINT  @  192,   "1950"
150 PRINT  @  256,   "1960"
160 PRINT  @  320,   "1970"
170 PRINT  @  384,   "1980"

200 REM    DRAWS THE ACTUAL GRAPHS
220 FOR X=12 TO 42
230    SET(X,6)  :  SET(X,7)
240 NEXT X

320 FOR X=12 TO 56
330    SET(X,9)  :  SET(X,10)
340 NEXT X

420 FOR X=12 TO 64
430    SET(X,12) :  SET(X,13)
440 NEXT X

520 FOR X=12 TO 102
530    SET(X,15) :  SET(X,16)
540 NEXT X

620 FOR X=12 TO  124
630    SET(X,18) : SET(X,19)
640 NEXT X
```

A shorter and more powerful graph generator can be written using
instructions we have not yet studied (see Lesson 13). In this version, the
years and the values to be plotted are stored in DATA statements and enter-
ed into the computer using a READ instruction.

```
  5 CLEAR 200
 10 CLS
100 PRINT @ 6, "GRAPH OF ENROLLMENTS AT THE UNIVERSITY OF OKLAHOMA "
110 READ Y, D
115 IF Y < 0 THEN 115
120 DATA 1940, 21, 1950, 28, 1960, 32, 1970, 51, 1980, 62, -1, -1
130 PRINT Y; STRING$(D,143)
140 GOTO 110
```

However, it is better to learn to walk steadily before one tries to run or
to fly—We invite you to use the above program either with the given data
or by changing instructions 100 and 120 to produce another histogram of
more interest to you. The end of your data should be indicated by -1,-1
as above. If the data values for D lie between 1 and 59, the program will
graph the data on a single line; otherwise more than one output line may
be required in which case you may wish to insert 135 PRINT in the above
program. Experiment a bit.

Try the small-flake snowstorm programs below.

```
100 SET(RND(127), RND(47))
110 IF RND(0)<.001 THEN CLS
120 GOTO 100
```

```
70 CLS
100 X=RND(127)
110 FOR Y=1 TO 43 + RND(3)
120    RESET(X,Y) :  SET(X,Y+1)
130 NEXT Y
140 GOTO 100
```

Here is another interesting program. The students claim it is a "campus planning program".

```
 50 N=1
 80 CLS
100 PRINT  "     CAMPUS PLANNING PROGRAM "; N
190 FOR K=1 TO 3+RND(17)
200    SX=RND(70)
205    SY=3+RND(23)
210    L=RND(55)
215    W=RND(20)
220    FOR X=SX TO SX+L
225       SET(X,SY) :  SET(X,SY+W)
230    NEXT X
235    FOR Y=SY TO SY+W
240       SET(SX,Y) :  SET(SX+L,Y)
245    NEXT Y
250    FOR Q=1 TO 100+RND(300) :  NEXT Q
255 NEXT K
260 FOR Q=1 TO 800  :  NEXT Q
265 N=N+1
270 GOTO 80
```

If you'd like to have your N-S walls the same width as the E-W walls, add:
243 SET(SX+1,Y):SET(SX+L+1,Y)

If you want a longer look at a special design on the screen, depress SHIFT @ to halt the program. Then depress any key, without SHIFT to continue.

Note instruction 260 FOR Q = 1 TO 800 : NEXT Q. This forces the program to count to 800 before it goes to instruction 80 and clears the screen. Separate instructions are separated by colons in lines 225, 240 250 and 260.

The following program contains several instructions we have not discussed yet. If you are particularly interested in learning about them,

consult Lessons 12 and 14. However, you can <u>use</u> the programs without having to understand LEN(Z$) or MID$(Z$,K,1).

```
100 CLS
110 Z$= "YOU HAVE ALREADY LEARNED LOTS, BUT STILL HAVE OTHER GOODIES
       TO INVESTIGATE, TOO."
120 K=0
130 E=INT((LEN(Z$)+1)/2)
140 FOR A=0 TO E STEP .5
150    X=15*SIN(A)
160    K=K+1
170    PRINT  TAB(X+30) ; MID$(Z$,K,1)
180    FOR Q=1 TO 50 :   NEXT Q
190 NEXT A
200 FOR Q=1 TO 300  :  NEXT Q
210 GOTO 100
```

Run the program. Then, change line 110 to

    110 Z$ = "_____"

where you type any message you wish between the quotes.

Let us write a program to create a rectangle on the video screen. Select as vertices the following points:

A(25,10)   B(45,10)
C(25,30)   D(45,30)

We might believe that the result will be a square, since the distances AB,BD,DC,CA each appear to be twenty units in length, i.e.

X-distances are each 45-25=20 units
Y-distances are each 30-10=20 units.

However, this is not valid. Let us write the program and then examine the output.

```
10 CLS
100 REM     DRAW LINE FROM (25,10) TO (45,10)
110 Y=10
120 FOR X=25 TO 45
130    SET(X,Y)
140 NEXT X

200 REM     DRAW LINE FROM (25,30)  TO (45,30)
210 Y=30
220 FOR X=25 TO 45
230    SET(X,Y)
240 NEXT X

300 REM     DRAW LINE FROM (25,10)  TO (25,30)
310 X=25
320 FOR Y=10 TO 30
330    SET(X,Y)
340 NEXT Y

400 REM     DRAW LINE FROM (45,10)  TO (45,30)
410 X=45
420 FOR Y=10 TO 30
430    SET(X,Y)
440 NEXT Y
```

RUN the above program.

OUTPUT

58

The rectangle was <u>not</u> a square. Indeed, it is about twice as tall as it is wide, since the "spots" (pixels) located at (X,Y) are actually rectangles (as you can see by typing SET(1ØØ,3Ø) [ENTER]). In other words, the X-direction units are only half as large as the Y-direction units.

To obtain a better approximation of a square, you could change each of the 45's to 65's (in instructions 12Ø, 22Ø and 41Ø). Change the steps as suggested and RUN the program again. It still isn't a perfect square, but it is much improved. What about lines 1ØØ, 2ØØ, and 4ØØ?

It is possible to make the lines of this square of equal weight by changing instructions 33Ø and 43Ø to

    33Ø SET(X,Y) : SET(X-1,Y)
    43Ø SET(X,Y) : SET(X+1,Y)

Try these alterations and see how the new program behaves.

It still does not produce a perfect square...perhaps because the rectangular "pixels" are really not <u>exactly</u> twice as high as they are wide.

Depress [BREAK] and [CLEAR] and type
LIST   [ENTER]

This will display your current program which should be:

```
 1Ø CLS
1ØØ REM    DRAW LINE FROM (25,1Ø) TO (65,1Ø)
11Ø Y=1Ø
12Ø FOR X=25 TO 65
13Ø    SET(X,Y)
14Ø NEXT X

2ØØ REM    DRAW LINE FROM (25,3Ø) TO (65,3Ø)
21Ø Y=3Ø
22Ø FOR X=25 TO 65
23Ø    SET(X,Y)
24Ø NEXT X

3ØØ REM    DRAW LINE FROM (25,1Ø) TO (25,3Ø)
31Ø X=25
32Ø FOR Y=1Ø TO 3Ø
33Ø    SET(X,Y) :  SET(X-1,Y)
34Ø NEXT Y

4ØØ REM    DRAW LINE FROM (65,1Ø) TO (65,3Ø)
41Ø X=65
42Ø FOR Y=1Ø TO 3Ø
43Ø    SET(X,Y) :  SET(X+1,Y)
44Ø NEXT Y
```

Your task is to adjust the width of the rectangle to produce the best square you can. Increasing the 65's in instructions 120, 220 and 410 slightly (say, to 75) may be all that is needed. Play with it a bit yourself.

Actually, the program can be shortened considerably by drawing the two parallel lines in the same FOR.....NEXT loop, as illustrated below.

```
NEW
  10 CLS
 100 FOR X=24 TO 73
 110    SET(X,10) :  SET(X,30)
 120 NEXT X

 200 FOR Y=11 TO 29
 210    SET(24,Y) :  SET(25,Y)
 220    SET(72,Y) :  SET(73,Y)
 230 NEXT Y
 250 PRINT  @ 405, "SQUARE" ;
 300 GOTO 300
```

Instruction 250 prints the word SQUARE in the drawn square. The ; at the end of instruction 250 prevents that instruction from clearing the following line and thus disrupting your square.

Instruction

    300   GOTO 300

puts the program in a "tightloop" that prevents the TRS-80 from displaying

```
READY
>_
```

which could disrupt the display. To get the computer out of this tight-loop, depress the BREAK key.

CHR$( )

 Each screen character or action (including scroll, backspace, car-
riage return, and convert to double-size letters) has a numeric code
that corresponds to it.

 In Level II BASIC (but not in Level I) it is possible to use the
instruction CHR$(*n*) (where *n* is an integer or a variable that has value
between 0 and 255) to obtain the screen character or action correspond-
ing to the number *n*.

```
100 INPUT K
110 PRINT K;
130 PRINT CHR$(K)
200 GOTO 100
```

will permit you to test this out.

 Use the following input values, among others, to obtain a variety
of symbols:

 33, 36, 42, 60, 88, 91, 92, 109, 134, 148, 191

 The following program will print the character number along with a
row of that particular character. This may be useful as a border on a
graphic display.

```
10 CLS
90 PRINT    "PLEASE TYPE NUMBERS BETWEEN 33 AND 191"
100 INPUT K
110 PRINT K,
120 FOR L=1 TO 45
130    PRINT CHR$(K);
140 NEXT L
150 PRINT
200 GOTO 100
```

Try 33, 36, 38, 42, 47, 60, 62, 72, 88, 91, 92, 109, 125, 126, 134, 135,
141, 148, 152, 187 and 191 to obtain a variety.

To see the entire array of useful display symbols, change instruction 100
and add instruction 170 as suggested below:

 100 FOR K=33 TO 191
 170 NEXT K

 After you have run the program a few cycles, you may wish to permit
K to range from 0 to 255 in instruction 100. But the effect of CHR$( )
values outside of the range 33 to 191 is harder to visualize from the

program. For example: CHR$(8) backspaces and erases character. The effect of CHR$(W) for several W-values is given below:

W-value	Effect or Character Printed
8	Backspaces and erases one character
10-13	Linefeed / Carriage Return
14	Turns on Cursor
15	Turns off Cursor
23	Converts to double-size letters
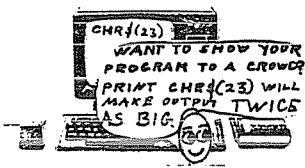
This is handy in your programs! Just include

1Ø8 PRINT CHR$(23)

and your output will be double-sized until the next CLS.

24	Backspace Cursor (←)
25	Advance Cursor (→)
26	Downward Cursor (Linefeed or ↓)
27	Upward Cursor (Linefeed or ↑)
28	"HOME", Returns Cursor to (Ø,Ø) position
29	Moves Cursor to beginning of Current Line
30	Erase to End of Line
31	Clear to End of Frame
32	Space
33	!
34	"
35	#
36	$
37	%

W-value	Effect or Character		W-value	Effect or Character
38	&		95	—— (Underscore)
39	/		96–127	Lowercase a–z (but TRS-80 does not display lowercase)
40	(			
41	)		128	Space
42	*		129–191	Various combinations of Rectangles for fast SET()
43	+			
44	,		192–255	Tabs for Ø to 63 spaces
45	–			
46	.			
47	/			
48	0			
49	1			
50	2			
51	3			
52	4			
53	5			
54	6			
55	7			
56	8			
57	9			
58	:			
59	;			
60	<			
61	=			
62	>			
63	?			
64	@			
65–90	A – Z			
91	↑			
92	↓			
93	←			
94	→			

GRAPHING FUNCTIONS

YES I CAN
COMPUTE
20+20 SIN(x)
AND EVEN MAKE
THE GRAPH.
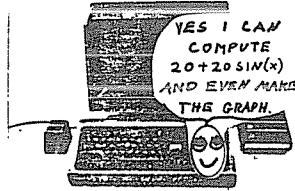
Just so you won't feel
this chapter is entirely friv-
olous, let's write a program
to graph the function

$y = 20+20$ sin$x$ for $0 \le x \le 3.2$
(radians, of course). If
you don't know what that
means, don't let it bother
you. The point is that the
computer knows what it means.
If you just realize that for each value of x between 0 and 3.2 that you
think of, the computer will supply you with a corresponding value of
$y = 20+20$ sin$x$, that's all you need to know. It doesn't really make any
difference that sin$x$ is frequently used in trigonometry and you don't
know any trigonometry...just hang on and have faith.

Let's write a short program to see that the computer really can supply the
desired value.

Level II BASIC

```
 90 PRINT "X", "Y"
100 FOR X=0 TO 3.2 STEP 0.1
110     Y=20+20*SIN(X)
120     PRINT X,Y
130 NEXT X
RUN
```

Your computer should produce:

X	Y
0	20
.1	21.9967
.2	23.9734
.3	25.9104
.4	27.7884
.5	29.5885
...	....

64

Now, what we want to do is to graph this function instead of making a table of values.

Recall that the X,Y coordinates on the screen are ($\emptyset$,$\emptyset$) at the top left and Y increases downward. This is inverted from the usual graph.

We shall create a new variable Y1 = 40 - Y so that

Y	Y1
$\emptyset$	4$\emptyset$
2$\emptyset$	2$\emptyset$
4$\emptyset$	$\emptyset$
...	...

This will invert our graph. (Think about that a bit if it bothers you.)

We'll also introduce a new variable X1 = 10*X to spread the function out on the screen. The program

```
9Ø CLS
1ØØ FOR X=Ø TO 12.5 STEP Ø.1
12Ø     Y=2Ø+2Ø*SIN(X)
13Ø     X1=1Ø*X
14Ø     Y1=4Ø-Y
15Ø     SET(X1,Y1)
16Ø NEXT X
17Ø GOTO 17Ø
```

will produce an acceptable graph of the function

$$y = 20 + 20 * SIN(x)$$

The last instruction   17Ø GOTO 17Ø   is inserted to keep the TRS-80 from displaying

READY
>_

which would spoil the graph. To get out of this "tight loop", depress   BREAK .

     Later, you can learn to make even fancier graphs that show the axes and units as well as the shape, but here we only introduce the basic idea.

The following program displays a series of interesting graphs.

```
1ØØ CLS
11Ø A=1+RND( 5 )
12Ø B=1+RND( 5 )
13Ø C=1+RND( 5 )
14Ø PRINT A,B,C
15Ø FOR X=Ø TO 12.7 STEP .1
16Ø    Y=25-5*(SIN(A*X) - SIN(B*X) + COS(C*X))
17Ø    SET( 1Ø*X,Y)
175    FOR Q=1 TO 2Ø : NEXT Q
18Ø NEXT  X
19Ø FOR Q = 1 TO 5ØØ : NEXT Q
2ØØ  GOTO 1ØØ
```

```
1Ø REM    ANOTHER "RANDOM ART"  PROGRAM
9Ø CLS
95 FOR L=1 TO 3Ø
1ØØ    FOR K=Ø TO 15
1Ø5      P=K*64 + RND(64) -1
11Ø        PRINT @  P, CHR$(128+RND(63));
12Ø    NEXT K
125 NEXT L
13Ø FOR Q=1 TO 9ØØ  :  NEXT Q
14Ø GOTO 9Ø
```

     After you have RUN the program a few times, add

  115 PRINT @ RND(1Ø23), CHR$(128+RND(63));

     and RUN it again.

A ninth-grade student, Anthony Tipton, from Millwood High School, Oklahoma City, Oklahoma, produced the following rather interesting program after reading a preliminary version of this chapter.

```
3 REM     ANTHONY D. TIPTON
5 REM     1717 NE 50TH
7 REM     OKC,OK  73111
9 REM     427-0137
99 CLS
100 FOR Z=1 TO 3  :  CLS
101    FOR X=0 TO 127
102       Y=23
103       SET(X,Y)
104    NEXT X
105    FOR Y=0 TO 47
106       X=63
107       SET(X,Y)
108    NEXT Y
170    FOR X= -6.2 TO 6.2 STEP .05
175       ON Z GOSUB 300, 400, 500
180       Y= -Y
190       X1=X*10  :  Y1=Y*5
200       A=X1+63  :  B=Y1+23
205       IF B<0 THEN 220
206       IF B>47 THEN 220
207       IF A>127 THEN 220
208       IF A<0 THEN 220
210       SET(A,B)
220    NEXT X
230 NEXT Z
250 GOTO 99

300 Y=SIN(X)
301 PRINT @   896,  "SINE";
302 RETURN

400 Y=TAN(X)
401 PRINT  @   896,  "TANGENT";
402 RETURN

500 Y=X*X -4
501 PRINT  @   896,  "PARABOLA";
502 RETURN
```

Sophisticated readers will see ways to speed up the program. However, it serves rather nicely at its current speed as a classroom demonstration illustrating how flat the sine curve is at max and min points, and how nearly linear where it crosses the x-axis, allowing for instructor comment on each curve.

67

SUMMARY OF LESSON 4

Let's see what you have learned thus far...

RND($\emptyset$)   Produces a random decimal value between 0 and 1, not includ-
          ing endpoints.

RND(n)   Produces a random integer between 1 and n, including end-
          points.

PRINT @ 96$\emptyset$, "NEXT VALUE IS"; N   Prints at the bottom of the screen
                                    (location 96$\emptyset$).

Run 1400 | ENTER | Begins running a program at statement 1400.

SET(X,Y)   Turns on rectangle at point (X,Y).

RESET(X,Y)   Turns off rectangle at point (X,Y).

POINT(X,Y)   Tests point (X,Y) to see if it is lighted or not (very
                useful in some games).

FOR Q = 1 TO 4$\emptyset\emptyset$: NEXT Q   The : separates different instructions
                                on the same line. This particular line is
                                a "time waster" to slow down the computer.

PRINT CHR\$(n)   Obtains the character or screen action de-
                                signated by n.

ON N GOTO a, b, c,...,z   When N takes on the value k, then pro-
                           gram branches to k^{th} statement number in
                           the list a, b, c,...,z. (See Problem 11,
                           Lesson 4.)

Mostly, we have learned to use some different "graphic" capabilities of
the computer for fun and for work.

PRACTICE SESSION 4

1. If you haven't tried a variation of the PRINT AT RND(1ØØØ),"HI____"
program suggested, do so now. That is how you increase your under-
standing of the BASIC language you are using. Try this one, too.

```
NEW  [ENTER]
5ØØ X=RND(125)
51Ø Y=RND(47)
52Ø SET(X,Y) :  SET(127-X,Y)
53Ø IF RND(Ø)<.ØØ2 THEN CLS
54Ø GOTO 5ØØ
```

2. Try the "snowflake" program given in the text.

```
9Ø CLS
1ØØ X=RND(127)
11Ø FOR Y=1 TO 43+RND(3)
12Ø   RESET(X,Y) :  SET(X,Y+1)
13Ø NEXT Y
14Ø GOTO 1ØØ
```

Then add

```
1Ø5 T=RND(127)
125 RESET(T,Y-1) :  SET(T,Y)
```

Type   LIST [ENTER] to see the new program.
Then, type   RUN   [ENTER] to run it.

3. Add a third snowflake to the extended program of Problem 2 by adding
appropriate instructions in 1Ø6 and 126. Try your program out.

4. Run the following program:

```
NEW  [ENTER]
1ØØ X=Ø
11Ø Y=Ø
12Ø SET(X,Y) :  SET(127-X,Y)
13Ø X=X+1
14Ø Y=Y+1
17Ø GOTO 12Ø
```

If you are using Level I BASIC, it should fill the screen with dia-
monds.
If you are using Level II BASIC, the program will "bomb off" whenever
Y>47 or X>127. This can be avoided by inserting:

```
15Ø IF Y > 45 THEN Y=RND(1Ø)
16Ø IF X > 125 THEN X=RND(15)          Try it again!
```

5. Try the following program on your TRS-80.

```
NEW    ENTER
 90 CLS
100 X=4+RND(120)
110 Y=3+RND(40)
120 SET(X+1,Y+2) :  SET(X,Y+2) :  SET(X+1,Y+1) : SET(X,Y+1)
130 SET(X-1,Y) :   SET(X-2,Y)
140 SET(X+2,Y) :   SET(X+3,Y)
145 SET(X,Y-1) :   SET(X+1,Y-1)
160 IF RND(0)<.02 THEN CLS
200 GOTO 100
```

Note: The programs of Problems 5 to 13 are somewhat related and may be placed on the TRS-80 without using NEW ENTER to save re-typing.

After it runs, try changing 160:

```
160 IF RND(0)<.2 THEN CLS
```

and then to

```
160 IF RND(0)<.005 THEN CLS
```

6. Extend and modify the program of Problem 5 by adding:

```
170 IF RND(0)<.7 THEN 100
200 X=0
210 Y=0
220 SET(X,Y) :  SET(127-X,Y)
230 X=X+1
235 IF X>120 THEN X=RND(6)
240 Y=Y+1
245 IF Y>45 THEN Y=RND(5)
290 IF RND(0)<.01 THEN 90
300 GOTO 220
```

Type RUN 200 ENTER. This will run the program from 200 on, to check it out. After it is checked out type RUN ENTER.

7. Run the following program:

```
1100 FOR X=32 TO 255
1110    PRINT X,
1120    FOR L=1 TO 45
1130       PRINT CHR$(X);
1140    NEXT L
1150    PRINT
1160 NEXT X
```

Run the program a couple of times. Then change statement 1100 to:

```
1100 FOR X=5 TO 35
```

and RUN the new program.

8. Try this program:

```
600 CLS
610 K=32+RND(150)
620 PRINT @ RND(1023), CHR$(K);
630 GOTO 610
```

After you have RUN the above program a few times, delete the semi-colon at the end of line 620 and RUN it again. Can you detect any difference? Why?

After RUNning the revised program (without the ;) add:

```
615 IF RND(0)<.01 THEN PRINT @ RND(1000),"HI your name";
```

and RUN it.

9. Wasn't that fun! Now, try this one:

```
500 X=RND(125)
510 Y=RND(47)
520 SET(X,Y) : SET(127-X,Y) : SET(X,48-Y) : SET(127-X,48-Y)
530 IF RND(0)<.002 THEN CLS
535 GOTO 500                    (Compare this with Problem 1.)
```

10. Here's another goodie.

FOR LEVEL II

```
700 FOR Q=1 TO 200 : NEXT Q : CLS
710 FOR X=15360 TO 16383
720    POKE X,191
730 NEXT X
760 RESET(RND(124)+1, RND(40)+2)
770 IF RND(0)<.001 THEN 700
780 GOTO 760
```

FOR LEVEL I

```
700 FOR Q=1 TO 200 : NEXT Q : CLS
710 FOR Y=0 TO 47
712    FOR X=0 TO 125 STEP 2
720       SET(X,Y) : SET(X+1,Y)
730    NEXT X
740 NEXT Y
760 RESET(RND(124)+1, RND(40)+2)
770 IF RND(0)<.001 THEN 700
780 GOTO 760
```

11. The following knits together the four separate programs which we
    tested in Problems 5, 6, 9, and 10. In each case, the individual
    programs have been modified to send control to instruction 800 every
    once in a while. Instruction 800 then sends control, at random, to
    one of the subprograms beginning at instruction 90, 100, 200, 500,
    700, 200, 500, 100.

```
  5 T=.01
 10 DEFINT X,Y      (Omit this instruction on Level I BASIC.)
 90 FOR Q=1 TO 200 :   NEXT Q  :  CLS

100 X=4+RND(120)
110 Y=3+RND(40)
120 SET(X+1,Y+2) :  SET(X,Y+2) :  SET(X+1,Y+1) :  SET(X,Y+1)
130 SET(X-1,Y) :  SET(X-2,Y)
140 SET(X+2,Y) :  SET(X+3,Y)
145 SET(X,Y-1) :  SET(X+1,Y-1)
160 IF RND(0)<.02 THEN CLS
170 IF RND(0)<.7 THEN 100
180 IF RND(0)<T THEN 800

200 X=0
210 Y=0
220 SET(X,Y) :  SET(127-X,Y)
230 X=X+1
235 IF X>120 THEN X=RND(6)
240 Y=Y+1
245 IF Y>45 THEN Y=RND(5)
295 IF RND(0)<T THEN 800
300 GOTO 220

500 X=RND(125)
510 Y=RND(47)
520 SET(X,Y) :  SET(127-X,Y) :  SET(X,48-Y) :  SET(127-X,48-Y)
530 IF RND(0)<.002 THEN CLS
535 IF RND(0)<T THEN 800                   On Level I BASIC use:

700 FOR Q=1 TO 200  :  NEXT Q  :  CLS   710 FOR Y=0 TO 47
710 FOR X=15360 TO 16383        )        712    FOR X=0 TO 125 STEP 2
720    POKE X,191               }        720       SET(X,Y) : SET(X+1,Y)
730 NEXT X                      )        730    NEXT X
760 RESET(RND(124)+1, RND(40)+2)         740 NEXT Y
770 IF RND(0)<.0001 THEN 90
775 IF RND(0)<.1*T THEN CLS  :  GOTO 800
780 GOTO 760

800 ON RND(8) GOTO 90,100,200,500,700,200,500,100
```

Instruction

800 ON RND(8) GOTO 90, 100, 200, 500, 700, 200, 500, 100

sends the program to the nth instruction in the list 90, 100, 200,
500, 700, 200, 500, 100, depending upon the value of RND(8) which
must be one of $n = 1,2,3,4,5,6,7,8$.

12.

```
390 CLS
400 X=30+RND(65) :  Y=10+RND(28)
410 SET(X,Y) :  SET(126-X,Y)
411 SET(X,46-Y) :  SET(126-X,46-Y)
415 X=X+RND(3)-2
420 X=X+RND(3)-2
430 IF X>125 THEN X= 125
435 IF X<1 THEN X=1
440 Y=Y+RND(3)-2
445 IF Y<1 THEN Y=1
450 IF Y>45 THEN Y=45
451 IF RND(0)<.009 THEN FOR Q=1 TO 500  :  NEXT Q  : CLS : GOTO 410
453 IF RND(0)<.01 THEN 400
455 IF POINT(X,Y)  THEN RESET(X,Y) :  RESET(126-X,Y) : GOTO 415
460 GOTO 410
```

Try the above program on your TRS-80. After you have enjoyed it a bit, undertake the following modifications. Is the design symmetric right to left? Is it symmetric top to bottom?

Add the following instruction:

```
413 FOR Q=1 TO 200  :  NEXT Q
```

which does nothing but slow down the action by forcing the computer to count to 200 each time it passes instruction 413. Run the above a bit before continuing...so you can follow the action in slow motion.

Try changing the .01 in instruction 453 to .05 and see how the pattern is affected.

Can you combine the result of instructions 415 and 420 into one instruction? Do so. (Note: Although the instructions appear identical, the values of X on which they work are not, so be careful.)

13. Modify the program of Problem 11 to also include the "Art" program of Problem 12, or your modification thereof.

14. Write an ART program of your own.

15. Write a program using the PRINT CHR$( ) instruction in some way.

16. Write a program to display an 8 by 8 chessboard (with black and white squares).

17. Write some programs of your own choice. You really do have the instructions needed for most programs.

18. Write a program that begins

    1ØØ X=RND(5Ø) : Y=RND(2Ø) : L=RND(15)

    and will then draw a square having upper left-hand corner (X,Y) and
    length of side L *y*-units. When your program is running well,
    enclose it in a FOR___NEXT loop so the program draws from 6 to 10
    squares. A possible over-program might be:

    1Ø CLS
    2Ø Z=RND(5)
    3Ø FOR K=1 TO (5+Z)

    1ØØ   X=RND(5Ø) : Y=RND(2Ø) : L=RND(15)

    Your square-drawing program here.

    5ØØ NEXT K
    51Ø FOR Q=1 TO 8ØØ : NEXT Q
    52Ø GOTO 1Ø

19. Write a program to draw a design of your choice: stairstep, circle,
    tree, box, face, robot, or whatever turns you on. However, choose
    your design first, then try to approximate it on the computer.

20. Make a bar graph showing data of your choice for several years.
    Note: scale your data so the longest bar ends before X = 125.

21. Make a cartoon of a dog that wags its tail on the display screen.

22. Make a smiling clown face on the display screen. Then expand your
    program so the clown winks or cries or talks or something that in-
    volves picture movement.

23. Create an animated "stick person" who walks across the screen, bows,
    and then continues walking offscreen.

24. Investigate some commercially available program that uses animation.
    Some possibilities are:

        Dancing Demon
        Life (Conway's simulation of cell growth)
        Android Nim
        Star Wars
        Chess programs

    See almost any of the magazines listed in Lesson 15 or consult your
    local computer store or Radio Shack.
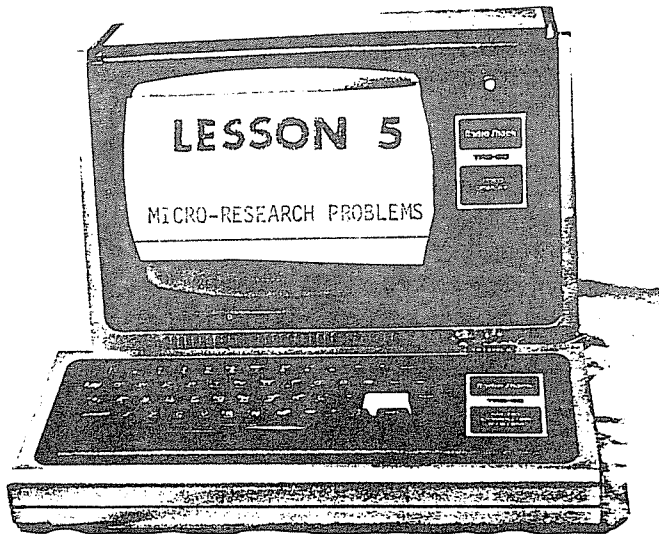
MICRO RESEARCH PROBLEMS

In Lesson 3, the number 30 Challenge Problem asks:

"The song 'Twelve Days of Christmas" mentions various gifts that
'my true love gave to me.' Let's interpret the song so that
'on the third day of Christmas, my true love gave to me

> Three French hens
> Two turtle doves
> And a partridge in a pear tree.'

My true love gave me six presents (3 French hens, 2 turtle doves,
and 1 partridge, if we don't count the pear tree as a gift). How
many gifts in total did my true love give to me during the twelve
days of Christmas? Write a program to determine the sum.
Answer: 364, if you do not count the pear tree as a gift."

A student submitted the following program in response to Challenge Problem 30. It does not produce the correct answer because the student
has violated a very fundamental programming rule. Can you find the error
and correct it?

```
 5 CLS
10 S=0
20 FOR K= 1 TO 12
30    S = S+K
40    PRINT K;
50 IF K= 1 THEN 90
60    K = K-1
70    GOTO 30
90    PRINT "    "; S
100 NEXT K
110 PRINT "    TOTAL = "; S
```

Before you continue, see what you can do to help our slipshod student. .

The basic principle our slipshod student violated is.

```
Do NOT change a  FOR...NEXT  variable

    inside of the FOR...NEXT loop.
```

If you look at the output from the program on the previous page

```
1           1
2    1      4
2    1      7
2    1      10
2    1      13
2    1      16
            etc.
```

since the first column in each line (separated by ; spacing) presumably contains the K values and the right most values (separated by , spacing) give the current S values, it is apparent that K is not taking on the values from 1 to 12 as might be expected in FOR K= 1 TO 12.

Indeed, the program blunder (changing the value of K inside the FOR...NEXT loop) forces K back to K=1 by the end of each loop, so the new K value, namely K+1, is always K=2.   Live and learn.

If you haven't worked on this Challenge Problem, you should before continuing. It is possible to make the "blunder" into a valid running program merely by changing instructions 2∅ and 1∅∅ and adding an instruction 25. See if you can do so before continuing.

Here is our new program, with new instructions at 2∅, 25, and 1∅∅ as indicated.

```
       5  CLS
      1∅  S=∅
☛     2∅  FOR L = 1 TO 12
☛     25     K = L
      3∅     S = S+K
      4∅     PRINT K;
      5∅  IF K = 1   THEN  9∅
      6∅     K = K-1
      7∅     GOTO 3∅
      9∅  PRINT "    ";S
☛    1∅∅  NEXT L
     11∅  PRINT "   TOTAL=";S
```

This produces the reasonably understandable output

```
RUN
 1   1
 2   1    4
 3   2   1    1∅
 4   3   2   1    2∅
 5   4   3   2   1    35
 6   5   4   3   2   1    56
 7   6   5   4   3   2   1    84
 8   7   6   5   4   3   2   1    12∅
 9   8   7   6   5   4   3   2   1    165
1∅   9   8   7   6   5   4   3   2   1    22∅
11  1∅   9   8   7   6   5   4   3   2   1    286
12  11  1∅   9   8   7   6   5   4   3   2   1    364

   TOTAL = 364

READY

>_
```

Mathematicians might use the Greek symbol sigma, Σ, to indicate the sum and express this problem as

$$\sum_{L=1}^{12} \left(\sum_{K=1}^{L} K \right)$$

Would the program listed on the following page achieve the same result?

77

```
200  SL = Ø
210  FOR L = 1 TO 12
220    SK = Ø
230    FOR K = 1 TO L
240      PRINT K;
250      SK = SK + K
260    NEXT K
270    PRINT "   "; SK
280    SL = SL + SK
290  NEXT L
300  PRINT "   TOTAL="; SL
```

Don't type NEW. Just start this program beyond where the first program ended, so you can run either one by using:

RUN [ENTER]

 [BREAK]

or

 RUN 200   [ENTER]

or get both, by merely typing

RUN [ENTER]

The final total is the same in each case (TOTAL =364) but the intermediate sums (at the extreme right of each line) are quite different.

Hmmm? How would you change the second program to get it to mimic the output of the first program? Well, we want line

    270 PRINT "   "; SK   to print the value of SL instead of SK.

Let's change 270 to

    270 PRINT "   "; SL

and rerun it.

At first, that may look better, but a little more careful examination shows that this revision of the second program produces a running (cummulative) sum that is always <u>one line late</u>. Hmmm?

If we interchange instructions 270 and 280, giving

```
200  SL = Ø
210  FOR L = 1 TO 12
220    SK = Ø
230    FOR K = 1 TO L
240      PRINT K;
250      SK = SK + K
260    NEXT K
270    SL = SL + SK
280    PRINT "   "; SL
290  NEXT L
300  PRINT "   TOTAL="; SL
```
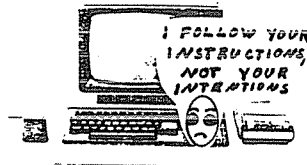

This program should now produce the same result as the program on the previous page. Does it? Yes, it does. Frankly, I prefer the program at

the top of the previous page that lists the number of gifts each day on the right edge, rather than the running total.

Actually, that problem is simple enough so that we could have computed the result without using a computer...but since a computer was readily available, it may have been easier to use the computer. Or it would have been, if we hadn't made so many foolish blunders in our program. However, correcting blunders is one way to learn to write blunder-free programs.

Let us turn our attention to some problems that are not easy to solve without a computer. The term micro-research problems seems appropriate for problems that are not to be found in most text books. The easiest way to find the answer to a micro-research problem may well be to solve it for yourself. Interestingly, you do have enough mathematical and computer programming ability, so that with the help of a TRS-80 micro computer, you can solve many problems that would have been beyond your ability and/or patience five lessons ago.

Let's get going!

Now that you can use your computer to solve a really difficult problem (obtaining roots of messy equations, Lesson 3) and have fun using the graphics capability of your TRS-80 (Lesson 4), it is time to think about some interesting micro-research problems on which you can expect to make reasonable progress using your computer. We shall walk through the solution of one quite difficult micro-research problem in detail - but you may wish to solve it yourself before reading on.

Problem:

Find perfect square integers like 5776 which have
the property that they end in their square roots. $\sqrt{5776} = 76$

Admittedly, this problem is not earthshaking, but it would not be very easy to solve without a computer. Try it yourself before continuing, with or without a computer.

STEPS FOR COMPUTER-ASSISTED PROBLEM SOLVING

Successful problem solving is apt to involve several quite different stages. Here is a series of steps that is sincerely recommended.

1. Be sure you understand the problem. Try to restate it in several ways. Then try to find a general method (algorithm) for solving the problem.

2. Examine a simple special case first.

3. Then, if it seems appropriate, program a computer to examine another special case.

4. Modify your computer program to examine a different special case.

5. Modify and generalize your computer program. Include tests to be sure the program is working as expected.

6. Run the new program and examine the output.

7. Now re-examine the output. Use your common sense and mathematical acumen to see if you can devise a better (faster and/or safer) algorithm or prove a theorem that will help solve the problem, unless it is already solved.

8. Go back to step 5, if the original problem has not yet been solved to your satisfaction.

This seems rather round-about at first, but experienced computer problem-solvers have found it is a much better technique than the usual technique of the amateur, who tries to start by writing the final program, or at least with the program of steps 5 and 6 right away. This is fine on trivial problems, but on more difficult problems it is really better to start easy and work up gradually, testing each stage.

Try it on our problem.

1. <u>Be sure you understand the problem.</u> Try to restate it.

   <u>Original Problem</u>

   Find perfect square integers like 5776 which have the property that they end in their square roots. $\sqrt{5776} = 76$

   <u>Restated Problem</u> (same problem, but from another viewpoint)

   Find non-negative integers N such that N^2 ends in N.
   Find integers $N \geq 0$ such that their squares $S = N^2$ end in the digits of N.

   The first technique (algorithm) that suggests itself may well be just to examine $S = N^2$ for each N and determine whether or not $S = N^2$ ends in N. Later you may be able to devise a better algorithm.

2. <u>Examine a simple special case first.</u>

   In attempting to solve any problem, it is well to examine several special cases before plunging in.

N	0	1	2	3	4	5	6	7	8	9
N^2	0	1	4	9	16	25	36	49	64	81

   So the one-digit numbers N whose squares end in N are N = 0,1,5,6.

3. <u>Write a computer program to examine another special case.</u>

   Let us write a program that will find the <u>three-digit</u> numbers N whose squares end in N. The last sentence displays two rather important problem-solving techniques:

   1. In computing, it frequently pays to rephrase a question by turning it around a bit.

   2. It is often helpful to examine a special case (here the 3-digit case) of a more general problem.

   You may wish to try writing such a program yourself before continuing.

   If you wish to examine the last 3 digits of a number S, then you need to devise a technique of finding the last 3 digits of a longer number. There is an easy way to do it using the INT (   ) function. INT (   ) produces the integer portion of the value inside of the parentheses. Thus, INT (34.761) = 34.

The instruction

T = S - 1000*INT (S/1000)

will produce the last three digits of S and store them as T. If this is not obvious, try it for 2 or 3 integral values of S.

For example: S = 123456
              S/1000 = 123.456
              INT(S/1000) = 123
              1000*INT(S/1000) = 123000
              S - 1000*INT(S/1000) = 123456 - 123000 = 456

Using this instruction, see if you can devise the desired program yourself.

    A program to examine the 3-digit numbers N such that S = N*N ends in N:

```
100   FOR N = 101 TO 999
110      S = N*N
120      T = S - 1000*INT(S/1000)
130      IF N<>T THEN 200
140      PRINT S;N
200   NEXT N
```

You may wish to run the above program before continuing.

The results are:
                 141376   376
                 390625   625

4. Modify your program to examine other special cases.

       Can you change the program to also obtain the 1-digit values of N whose squares end in N? -- namely, 0,1,5,6 as we saw when we worked the problem by hand?

       Of course, you can. Merely change instruction,

       100 FOR N = 101 To 999   100 FOR N = 0 TO 9
and   to
       120 T = S - 1000*INT(S/1000)   120 T = S - 10*INT(S/10)

and RUN the program.
Try it yourself.

       What changes would you make in the program to obtain the 2-digit values of N such that $S = N^2$ ends in the digits of N?
Please do so before continuing.

Although our current attack is clumsy, even a clumsy attack is often better than no attack at all. Let us now try to produce a program which will do the modifications for us. This is unnecessary in this particular problem, since the modifications are easily done, but we may learn some useful tricks along the way. Examine the following program and be sure you see how it works before running it.

5. <u>Generalize your computer program to examine several cases using</u>
    <u>the same program.</u>
    <u>Our new fundamental program</u>:

```
 1Ø   INPUT P
1ØØ   FOR N = 1+P/1Ø TO P-1
11Ø     S=N*N
12Ø     T=S-P*INT(S/P)
13Ø     IF N<>T THEN 2ØØ
14Ø     PRINT S;N
2ØØ   NEXT N
```

(where P is the integer 1, followed by as many zeros as there are digits in N)

The program seems to work well. If there is a way to change P from 1Ø to 1ØØ to 1ØØØ etc. as the program goes along, you could then use a slight modification of the old program. Try it yourself before continuing.

In any event, the modified program given below sounds as if it would work:

```
 1Ø   P=1Ø
1ØØ   FOR N = 1 + P/1Ø TO P-1
11Ø     S=N*N
12Ø     T=S-P*INT(S/P)
13Ø     IF N<>T THEN 2ØØ
14Ø     PRINT S;N
2ØØ   NEXT N
21Ø   P = P*1Ø
22Ø   GOTO 1ØØ
```

Try it...it works! Well, it works for a while anyway.

It really does need two more instructions

```
 9Ø   PRINT "NOW WORKING ON N<" ; P          Also change 22Ø  GOTO 9Ø.
```

This will tell you the general range of N-values on which the program is working. If it gets stuck, you at least know where. It will also give you estimates on how long it takes for each set of k-digit N values, for k = 1,2,3,.... Common sense suggests 3-digit numbers should take about 10 times as long as 2-digit numbers, since there are 10 times as many of them.

Remember to include another important check.
Any computer has numbers S so large it can-
not tell S from S+1. If a computer carries
only six digits of accuracy, for example,
and S is a 7-digit number, say

(True S) = 7654321

The computer shows this as 7.65432E+06,
which is read 7.65432 $*10^6$ or
7654320. Now (True S+1) = 7654322.
However, if your computer stores only
six decimal digits of accuracy, then

S+1 = 7.654322 $*10^6$

which is chopped off (or possibly rounded)
to six significant digits, giving

7.65432E + 06

which is precisely what you had for S.
Thus, the computer cannot distinguish
between S and S+1.

> IF IT'S VERY BIG I CAN'T DISTINGUISH S FROM S+1
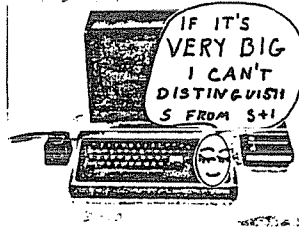
In this case, the <u>final digit</u> of S is no longer known. The computer
substitutes zero for it, and cannot tell if S ends in the same digits as
does N. Surely, you wish your program to STOP automatically if such a
crisis should arise (and it does, inevitably).

Do this by inserting the instruction

115   IF S=S+1 THEN PRINT "OVERFLOW ERROR ON S,N="; S;N : STOP

If the computer cannot distinguish between S and S+1, it will print
the requested message and then STOP. Otherwise, the program will ignore
instruction 115 and pass on to the next instruction.

The expanded program now reads:

```
 10   P = 10
 90   PRINT " NOW WORKING ON N< " ; P
100      FOR N = 1 + P/10 TO P-1
110      S = N*N
115      IF S = S+1 THEN PRINT "OVERFLOW ERROR ON S,N="; S;N : STOP
120      T = S-P*INT(S/P)
130      IF N<>T THEN 200
140      PRINT S;N
200   NEXT N
210   P=P*10
220   GOTO 90
```

Let's return to "Steps for Computer-Assisted Problem Solving".

6. Run the general program and examine the output.

Give it a try on your own computer before continuing.

The output is:

LEVEL II BASIC		LEVEL I BASIC	
. . . .		. . . .	

NOW WORKING ON N<1∅

25	5
36	6

NOW WORKING ON N<1∅∅

625	25
5776	76

NOW WORKING ON N<1∅∅∅

141376	376
390625	625

NOW WORKING ON N<1∅∅∅∅

OVERFLOW ERROR ON S,N = 3.35588E+∅7
                                                              5793

BREAK IN 115

Notice, our program "bombed off" at P=10000 with S,N= 3.35588E+07, 5793 as a result of the test in instruction 115.

NOW WORKING ON N<1∅

25	5
36	6

NOW WORKING ON N<1∅∅

625	25
5776	76

NOW WORKING ON N<1∅∅∅

141376	376
390625	625

NOW WORKING ON N<1∅∅∅∅

OVERFLOW ERROR ON S,N = 1.67772E+∅7
                                                              4096

Notice, our program "bombed off" at P=10000 with S,N = 1.67772E+07, 4096 as a result of the test in instruction 115.

This seems to reach the limits of exploration using LEVEL I BASIC unless we are willing to construct our own special "multiple precision" routines.

It is time to consider a new concept in computing:
DOUBLE PRECISION ARITHMETIC.

If you have LEVEL II BASIC on your TRS-80, you can also investigate the problem for 4-digit and 5-digit values on N (which produce 8 and 10-digit values of S) by including the instruction

$$5 \quad \text{DEFDBL} \quad S,P \qquad (\text{which is read "Define Double".})$$

which produces double length (16-digit) values for any variable beginning with S or P. (Maybe we can even run the program for 6,7, and 8-digit N-values which have 12,14, and 16-digit squares — wait and see.)

However, troubles appear:

The TRS-80 does not produce a double length product if single length numbers are multiplied together. Thus S = N*N will not produce a valid 8-digit square, if N is a single precision 4-digit number

$$\text{For } N = 9376 \qquad (\text{TRUE } S) = N^2 = 87909376$$

However, the computer rounds this to six digits and produces

$$N*N = 8.79094E + 07$$

which is stored in S as 87909400, rather than the correct value, 87909376.

The "obvious" solution is to make all three of S,P, and N double precision by using

$$5 \quad \text{DEFDBL} \quad S,P,N$$

However, the

$$\text{FOR } N = 1 + P/10 \text{ TO } P-1$$

instruction will not accept a double precision variable for N.

This is easily fixed by using

```
100   N = 1+P/10                          100   FOR N = 1+P/10 TO P-1
130   IF N<>T THEN 150    in place of     130   IF N<>T THEN 200
150   N = N+1
200   IF N<P THEN 110                      200   NEXT N
```

The program now reads:

```
  5  DEFDBL S,P,N
 1Ø  P = 1Ø
 9Ø  PRINT " NOW WORKING ON N<";P
1ØØ  N = 1 + P/1Ø
11Ø  S = N*N
115  IF S = S+1 THEN PRINT "OVERFLOW
     ERROR ON S,N="; S; N:STOP
12Ø  T = S - P*INT(S/P)
13Ø  IF N<>T THEN 15Ø
14Ø  PRINT S;N
15Ø  N = N + 1
2ØØ  IF N<P  THEN 11Ø
21Ø  P = P*1Ø
215  PRINT
22Ø  GOTO 9Ø
```

(Note→) COMMENTS

5 Omit instruction 5 if you are using LEVEL I BASIC.

1Ø The number of zeros in the current value of P is the number of digits in N.

9Ø It is good practice in a program whose output may be scarce to include a statement displaying what the program is doing.

115 This is a necessary precaution, when the computer cannot tell the difference between S and S+1, the program will STOP and so indicate on the screen.

21Ø Changes P to the next higher power of 10.

22Ø Repeats program with new P value.

The output is:

Level II BASIC

NOW WORKING ON N <1Ø

```
        25      5
        36      6
```

NOW WORKING ON N <1ØØ

```
       625     25
      5776     76
```

NOW WORKING ON N <1ØØØ

```
     141376    376
     39Ø625    625
```

NOW WORKING ON N <1ØØØØ

```
  879Ø9376    9376
```

NOW WORKING ON N <1ØØØØØ

```
 821289Ø625  9Ø625
```

NOW WORKING ON N <1ØØØØØØ
```
  119631Ø9376      1Ø9376
  793212889Ø625     89Ø625     etc.
```

Level I BASIC
(ommitting instruction 5)

NOW WORKING ON N< 1Ø

```
        25      5
        36      6
```

NOW WORKING ON N <1ØØ

```
       625     25
      5776     76
```

NOW WORKING ON N <1ØØØ

```
     141376    376
     39Ø625    625
```

NOW WORKING ON N <1ØØØØ

OVERFLOW ERROR ON S,N   1.67772E+Ø7
                                4Ø96

87

You will notice that for each new P value, it requires <u>ten times</u> as much computer time as for the previous P value. Run the program again and time it.

Level II BASIC	Level I BASIC

Your program did not "bomb out" like Level I did. But it sure can use up lots and lots of computing time.
To get from

You were stopped during
P=10000
at which time the instruction

P=100   to P=1000   took about 1.5 minutes
P=1000   "   P=10000   "" 15 minutes
P=10000   "   P=100000   "" 2.5 hours

115   IF S=S+1   THEN PRINT "OVERFLOW ERROR ON S,N=";S;N:STOP

halted the run. You may wish to delete instruction 115 and rerun the program to see what happens.

If you let your TRS-80 run another 24 or 48 hours, from P=100000 to P= 1000000, you will also find

Remember you can always stop the program by depressing   BREAK
You may then learn the values it was working on by typing

11963109376   109376
793212890625   890625

PRINT P,N,S,S+1   ENTER

and who knows what else???

NOW IS THE TIME FOR US TO GET SMART.

This program really works the problem in a very crude brute-force way. We are just examining every number, N. The number of numbers to be examined increases sharply as P increases.

	No. of cases	Approximate time
1 < P <10	8	Less than 1 second
10 < P <100	88	9 sec
100 < P <1000	898	90 sec = 1.5 min.
1000 < P <10000	8998	15 min.
10000 < P <100000	89998	150 min = 2.5hr.
100000 < P <1000000	899998	24 hr.
1000000 < P <10000000	8999998	9 or 10 days

Clearly we can reasonably expect to examine P < 10, 100, 1000, 10000, and even 100000, but after that, the computer time involved becomes exorbitant. Of course, you may reason that the TRS-80 might just as well be working on your problem as be sitting idle. It takes very little power. (It uses about as much power as a small light bulb, if you turn off the CRT display tube, which you might as well do if you are not watching it. You can turn it on and recapture whatever would have been on the screen at that moment. The computer continues to compute even when the display tube is turned off.)

Returning to our "Steps for Computer-Assisted Problem Solving",

7. Re-examine the output. Use your common sense and mathematical knowledge to devise a better (faster) algorithm to help solve the problem.

We really should have recognized at the beginning that N^2 must end in 0, 1, 5, or 6 if N^2 ends in N. This would have saved almost 60% of our computing time. Actually if $N > 9$, then N cannot end in 0 since then N^2 would end in 00, etc. Similarily if $N > 9$, N cannot end in k1 since then N^2 would end in (2k) 1 since $(10k+1)^2 = 100k^2 + 20k+1$. Similar mathematical arguments show that if $N > 9$ and if N^2 ends in N, then N ends in 5 or 6. So instead of increasing N by 1 each time we can jump by 10 and test both N and (N+1) to see if either has the desired property. The following program will cut 80% of our running time---doing an hour's testing in 12 minutes.

YOU ARE CLEVER!
THIS WILL CUT
MY WORKLOAD.
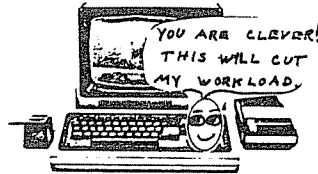
```
  5   DEFDBL S,P,N
 10   P=10
 90   PRINT "NOW WORKING ON N<";P
100   N = 5 + INT(P/100)*10
110   S = N*N
115   IF S = S + 1 THEN PRINT "OVERFLOW ERROR ON S, N =" S;N:STOP
120   T = S - P*INT(S/P)
130   IF N <>T THEN 150
140   PRINT S;N
150   N1 = N+1
152   S1 = N1*N1
154   T1 = S1 - P*INT(S1/P)
156   IF N1 <> T1 THEN 160
158   PRINT S1;N1
160   N=N+10
200   IF N <P  THEN 110
210   P = 10*P
215   PRINT
220   GOTO 90
```

Try it. It works, and we really might have been smart enough to have thought of discarding 80% of our unsuccessful cases in the beginning.

However---now that we have some output to look at, it may be possible to find an even more clever way to speed up our program. Look at the output. Do you notice anything special?

N²	N
25	5
36	6
625	25
5776	76
141376	376
39Ø625	625
879Ø9376	9376
8212 89Ø625	9Ø625
11963 1Ø9376	1Ø9376
793212 89Ø625	89Ø625

Two items demand our attention:

I.   When we investigated the one-digit N's by hand and on the first program (modified to numbers from Ø to 9), we found

N²	N
Ø	Ø
1	1
25	5
36	6

but our current computer program missed the first two values Ø, 1.
    Just why did that happen, and do you think that it missed any other values between P = 1Ø and P = 1ØØØØØØ? Are You Sure?

    We did obtain all the values between P = 1Ø and P = 1ØØØØØØ, but did miss N = Ø and 1 because this program did not test N = Ø or N = 1, since it started at N = P/1Ø + 1 for P = 1Ø which is 2.

    If you didn't notice that earlier, you need more computing experience before you can be considered a qualified amateur.

II.   Notice: The only large values of N such that N² ended in N were values of N that themselves ended in those exact digits for which we found some previous satisfactory N. Now that may be a real clue!

Consider the case starting at P=1$\emptyset\emptyset\emptyset$. We found

N^2	N
141376	376
39$\emptyset$625	625

It seems reasonable to believe that any 4-digit value of N whose square ends in N will have the value of N itself ending in either 376 or 625 since otherwise, N^2 would not end in N. Hmmm. Well, yes -- that seems reasonable.

However, some things that "seem reasonable" turn out <u>not</u> to be true. For example, one of my students extended the above conjecture by asserting, "Since there is only one <u>4-digit value</u> of N such that N^2 ends in N, there can be only one such N having k digits for any k>4."

However,

N = 1$\emptyset$9376 N^2 = 11963 1$\emptyset$9376

N = 89$\emptyset$625 N^2 = 79321289$\emptyset$625

blows that "reasonable conjecture".

(Why?) Well, because the student didn't recognize that the six-digit 1$\emptyset$9376 was a suitable extension of the four-digit N = 9376 under our original guess, with $\emptyset$ as the fifth digit. Hmmm.

Well, how about the original guess? If we could show that

> ### Conjecture
>
> If a given N^2 ends in the digits N $\geq$ 1$\emptyset$, then N itself must end in the same digits as some smaller such N.
>
> Not yet proved.

Would it be helpful in our search?

Well, you just bet it would be!

For example, in examining the 5-digit N values instead of examining 89998 cases, we would only need to examine the 20 cases X9376, X0625 (and possibly the 10 more cases X0376 unless we can show that is also impossible.... which we can).
I'm not going to prove the conjecture--but it is true.

To cut down from 89998 cases to 20
cases for the 5-digit N values is a
savings of 99.98% of the numbers to be
tested. The 6-digit values produce
even greater savings. What would for-
merly have taken twenty-five hours
should now run in less than a minute,
even if each number evaluated took
several times as long to test (since
the program may be messier than before).
That is indeed a worthwhile savings.
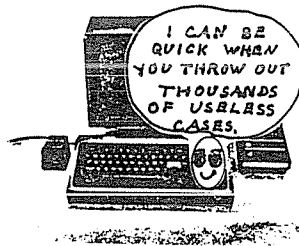
For now let us assume the slightly stronger result:

> If N^2 ends in N, then N ends in the digits of some smaller K
> such that K^2 ends in K. Furthermore, the previous K will be
> a K with one fewer digit than N, with the possibility of lead-
> ing zeros on a smaller K such as 9Ø625 being the N for K =
> Ø625, K^2 = 39Ø25.

Let's start our program with 3-digit N values. This will enable us to
test the program and see if it is running as we believe it should.

Store the "last two successful values of N" that we are trying to extend
in locations A and B. Starting with the 3-digit values of N, there will
be

    P = 1ØØØ
    A =   25   (since when N = 25, N^2 = 625)
    B =   76   (since when N = 76, N^2 = 5776)

to properly "seed" our program with the successful 2-digit values to be
extended.

Continuing with "Steps for Computer-Assisted Problem Solving," we find:

8. Go back to step 5.

5. Modify and generalize your computer program. Include tests to be
    sure the program is working as expected.

6. <u>Run the new program and examine the output</u>.

     The following program seems to produce the desired results in
moderate time. Study it until you understand it well enough to
explain it to one of your classmates who needs help.

```
  5   DEFDBL  S,P,N,A,B         (Omit 5 if using LEVEL I BASIC.)
 10   P=1000
 20   A=25
 30   B=76

 90   PRINT    "     WORKING ON N  <  "  ;  P
 95   Q=P/10

 99   REM      THIS BLOCK EXTENDS  A   VALUE
100   FOR K=1 TO 9
105   N = K*Q + A
110   S = N*N
115   IF S=S+1   THEN PRINT "OVERFLOW ERROR ON S,N = ";S;N:  STOP
120   T = S - P*INT(S/P)
130   IF N<>T  THEN 200
140   PRINT S;N
150   A=N
160   GOTO 300
200   NEXT K
210   REM         END OF BLOCK THAT EXTENDS  A   VALUE

250   REM

299   REM       THIS BLOCK EXTENDS  B   VALUE
300   FOR K=1 TO 9
305   N = K*Q + B
310   S = N*N
320   T = S - P*INT(S/P)
330   IF N<>T  THEN 400
340   PRINT S;N
350   B=N
360   GOTO 500
400   NEXT  K
401   REM         END OF BLOCK THAT EXTENDS  B   VALUE

450   REM

500   P = P*10
520   GOTO  90
```

Put it on your computer and see how it runs.

Output from <u>Level II BASIC</u>

```
    WORKING ON N < 1000
          390625              625
          141376              376

    WORKING ON N < 10000
         87909376            9376

    WORKING ON N < 100000
        8212890625          90625

    WORKING ON N < 1000000
       793212890625        890625
        11963109376        109376

    WORKING ON N < 10000000
      8355712890625        2890625
     50543227109376        7109376

    WORKING ON N < 100000000
    166168212890625       12890625
    7588043387109376      87109376

    WORKING ON N < 1000000000
OVERFLOW ERROR ON S,N =1.704786682128906D+17          412890625

BREAK IN 115

READY
>_
```

Output from <u>Level I BASIC</u>

```
    WORKING ON N < 1000
          390625              625
          141376              376

    WORKING ON N < 10000
OVERFLOW ERROR ON S,N = 2.13906E + 07       4625
```

Now we need to ask if the original problem has been solved.
If it was to <u>find</u> <u>all</u> perfect squares N^2 such that N^2 ends in the N,
then it is only partially solved. The program found all such N < 412890625
but there is no reason to suspect that is all there are. There is also
the possibility our program may be defective and may have overlooked some
N values less than 10^9 (After all - we did overlook 0 and 1 - remember?).

This is one way research is done. First, investigate a few special
cases. Next, use a simple program to investigate a few more cases. If
the results seem worthwhile, automate your program and let it run a bit.
Next, examine the output, make some conjectures and prove or disprove

their validity. Then, devise a new technique (algorithm) for carrying out the investigation and implement it.

Now you are ready to try to prove the most general case. It may or may not yield to your effort – but in any case, you have a pocketful of results that were previously unknown. We shall not attempt to go further here, but strongly suspect some able student will do so.

There is one additional check that should be made.

Did our last two programs produce the same results as far as they overlapped? If not, why not? This is important.

The two programs are:

<u>Program from page 89</u>

```
  5   DEFDBL   S,P,N
 1Ø   P = 1Ø
 9Ø   PRINT "NOW WORKING ON N<";P
1ØØ   N = 1 + P/1Ø
11Ø   S = N*N
115   IF S = S+1 THEN PRINT "OVERFLOW
      ERROR ON S,N=";S;N:STOP
12Ø   T = S - P*INT(S/P)
13Ø   IF N<>T  THEN 15Ø
14Ø   PRINT S;N
15Ø   N = N + 1
2ØØ   IF N<P  THEN 11Ø
21Ø   P = P*1Ø
215   PRINT
22Ø   GOTO 9Ø

RUN
```

<u>Faster program from page 93</u>

```
  5   DEFDBL   S,P,N,A,B
 1Ø   P = 1ØØØ
 2Ø   A = 25
 3Ø   B = 76
 9Ø   PRINT "    WORKING ON N<";P
 95   Q=P/1Ø
 99   REM  THIS BLOCK EXTENDS A VALUE
1ØØ   FOR K=1 TO 9
1Ø5   N = K*Q  + A
11Ø   S = N*N
115   IF S=S+1 THEN PRINT "OVERFLOW
      ERROR ON S,N=";S;N:STOP
12Ø   T = S - P*INT(S/P)
13Ø   IF N<>T  THEN 2ØØ
14Ø   PRINT S;N          .
15Ø   A=N
16Ø   GOTO 3ØØ
2ØØ   NEXT K
21Ø   REM  END OF BLOCK THAT EXTENDS
      A  VALUE
25Ø   REM
299   REM   THIS BLOCK EXTENDS B VALUE
3ØØ   FOR K=1 TO 9
3Ø5   N = K*Q + B
31Ø   S = N*N
32Ø   T = S - P*INT(S/P)
33Ø   IF N<>T  THEN 4ØØ
34Ø   PRINT S;N
35Ø   B = N
36Ø   GOTO 5ØØ
4ØØ   NEXT K
4Ø1   REM  END OF BLOCK THAT EXTENDS
              B  VALUE
45Ø   REM
5ØØ   P = P*1Ø
52Ø   GOTO 9Ø
RUN
```

The two rather different programs seem to agree as far as they go, but the longer program produces more results in two minutes, than the old program did in 48 hours or even a week. That is what is meant by effective programming.

Actually all values of N such that N^2 ends in N for N of 100 or fewer digits are known, but their investigation uses more mathematical sophistication than we are willing to impose here.

WHAT WE HAVE LEARNED IN LESSON 5.

The most important thing in Lesson 5 is the realization that you do already have enough computer adroitness to successfully undertake the investigation (and at least partial solution) of problems so difficult that a month ago you wouldn't even have tried them. You should also have discovered that it is frequently a nontrivial (but perfectly possible) task to program a computer to help solve research level problems.

Most important of all – you have discovered one of the fundamental truths of computer programming.

> When you have a program written and it is debugged and running properly, then if you reflect on the problem you will frequently find a better way to do it. If the problem is major, this is the time to think hard, then scrap your earlier effort and start over.

Of course if the problem is a "one time only" problem and if your existing program produces the desired results while you are thinking, you can forget the reprogramming unless you want to show the program to someone else or to use it again later.

SOME MICRO-RESEARCH PROBLEMS
YOU MAY UNDERTAKE

Don't expect all of these problems to be easy, or even solvable with your present knowledge. They are micro-research problems for your consideration. If you do more than two problems, you are to be congratulated. After working on a problem as stated, see what extensions and modifications you can devise; work on them.

1. If the 6's are cancelled in $16/64 \to 1\emptyset/\emptyset4 = 1/4$ the result is correct. $(26/65 \to 2\emptyset/\emptyset5 = 2/5$ is another example.) Find all examples of proper fractions such that AB/BC = A/C with A ≠ B where A,B,C are digits between $\emptyset$ and 9 inclusive.

2. If N! = 1*2*3*...*(N-1)*N   for N ≥ 1

   and $S(K) = 1! + 2! + 3! + 4! + \ldots + K! = \sum_{N=1}^{K} (N!)$,

   for what values of K ≥ 1 is S(K) a perfect square?
   (This requires some easy mathematical thinking after you have written and run a computer program.)

3. Write a program to accept an integer N > 2 as input and then type out N and the factors of N.

4. Let your input be three positive numbers A,B,C (not necessarily integers).
   a) Determine whether or not A,B,C can be the sides of a triangle. *(Note: If A,B,C = 1,5,3, the answer is "no".)*
   b) If A,B,C are acceptable as sides of a triangle, extend your program to also print the area of the triangle.
   c) If A,B,C do determine a triangle, extend your program to also compute the angles of the triangle.

5. a) Write a program that will accept a positive integer N as input and determine whether or not N is a prime. *(Note: 1 is not a prime. An integer N > 1 is prime if its only positive factors are 1 and N.)* Compare your program with those written by your classmates. Test it for several values of N > $6\emptyset\emptyset\emptyset$. See whose runs fastest, and why.
   b) Make a table of primes < $5\emptyset\emptyset\emptyset$.
   c) Extend your program to print out all the prime factors of N.

6. Write a program to accept two positive integers M and N and to print out M, N and also the largest positive integer which divides both M and N. Such an integer is called the greatest common divisor and may be found using a technique called Euclid's Algorithm or by simply trying all positive integers from the smaller of M and N down to 1 until you find a value which will divide both.

If you use the latter (inefficient, but effective) technique, your
program could begin:

```
1ØØ   INPUT "M,N =" M,N
1Ø6   IF M<=Ø  THEN 1ØØ
1Ø8   IF N<=Ø  THEN 1ØØ
1Ø9   REM:  INSTRUCTION  11Ø SETS  S = SMALLER OF (M,N) FOR USE IN 2ØØ
11Ø   IF M>N  THEN S = N ELSE S = M
2ØØ   FOR D=S TO 1  STEP -1
       .      .
       .      .
       .      .
```

7. Find two consecutive integers whose squares each use exactly the same
   digits (in a different order, of course). Thirteen and fourteen are
   such a pair; $13^2 = 169$ uses the same digits as $14^2 = 196$. Similarly,
   $157^2 = 24649$ and $158^2 = 24964$. Find other <u>pairs of consecutive inte-</u>
   <u>gers</u> whose squares are composed of the same digits.

8. a) Write a computer program to accept three values M,D,Y and deter-
      mine whether or not they are acceptable values for Month, Day,
      Year in that order. If $Y < 1590$ you may prefer to state the
      date is unacceptable rather than consider the various calendar
      reforms that took place prior to 1590.
   b) Extend the above program to accept two sets of three numbers and
      if both are acceptable as Month, Date, Year values, determine
      the number of days between them including both dates in your
      count.
      Example: 2,2Ø,1978 to 2,22,1978 is 3 days.

9. For the <u>distinct</u> digits A, B, C, D, E, F, G, H, I, J the product

   $$A*(BCDE) = FGHIJ$$

   has solution $4*(7039) = 28156$ and <u>a dozen</u> other <u>solutions</u>. Find them.

10. Write a program to accept a positive integer K and print it out and
    also print out the integer obtained by reversing the digits of K.
    Example: If K = 12345,
         print out: 12345   54321

11. The sequence
        1 2 3 4 6 8 9 12 16 18 24 27 32 ...
    is made by listing numbers of the form $N = 2^k \cdot 3^m$
    (with k and m integers $k \geq \emptyset$, $m \geq \emptyset$) <u>in order of increasing size</u>.
    It is not easy to devise an algorithm to generate and print them <u>in</u>
    <u>increasing order</u>, but that is only part of your task. The even more
    difficult problem is to find the 1ØØØth term of the sequence, or more
    generally, given an integer T, find the T-th term of the sequence.
    (Thanks to Fred Gruenberger for suggesting this problem.)

12. Write a program that will draw a random maze on the screen such that it is possible to reach every cell of the maze, but there is only one "best" path (i.e. without retracing) through the maze. Then, extend your program so that the computer will "solve" the maze it created.

13. a) Determine the sum of the N-digit numbers that can be formed using N specified digits for each possible set of 2,3,4 and 5 digits.
    Examples:

    N = 2   Digits = 3,8   Then sum = 38 + 83 = 121
             Digits = 4,1   Then sum = 14 + 41 = 55   (Yes, we count
             Digits = 2,2   Then sum = 22 + 22 = 44   22 both ways.)

    N = 3   Digits = 1,2,3   Then sum =
                              123 + 231 + 312 + 132 + 321 + 213 = 1332
             Digits = 3,8,9   Then sum =
                              389 + 893 + 938 + 983 + 839 + 389 = 4440

    b) Notice that the sum seems always to be divisible by 11. Can you prove this in general? What other factors will the sum have?

    c) Can you show that an N-digit number will have N! different permutations, (N-1)! of which start with each digit, and (N-1)! of which have each digit in the second place, etc.?

    d) If you prove each of the above, you may also be able to show that the sum of the N! numbers obtained as permutations of the N digits will always be (N-1)! * (sum of the original digits) * (the integer consisting of N ones). For N = 8, Digits = 1,2,3,4,5,6,7,8 there are 40320 integers and their sum is 2015999979840.

    e) What happens if a number containing repeated digits is counted only once? Thus N = 2, Digits = 2,2 Sum = 22   (not 22 + 22 = 44). [This is a rough problem.]

14. A palindrome is a phrase or sentence which reads the same forward as backward.

    "Too Hot To Hoot"
    "Ma Is As Selfless As I Am"
    "Retracting, I Sign It, Carter"
    "Marge Lets Norah See Sharon's Telegram"

    There are also numbers which are palindromes. It is fairly easy to find perfect squares which are also palindromes.

    $(111)^2 = 12321$
    $(202)^2 = 40804$
    $(264)^2 = 69696$

It is possible to prove that there exist infinitely many perfect squares that are palindromes. However, perfect squares that are palindromes having an <u>even number of digits</u> such as $(836)^2 = 698896$ are fairly rare.

Investigate the subject of perfect squares that are also palindromes.

15. For what values of $N > 1$ does $W = 2^N - 1$ contain a factor > 1 which is a perfect square?

If $N = 6$, $W = 2^6 - 1 = 63 = 3^2 \cdot 7$

If $N = 7$, $W = 2^7 - 1 = 127$ which does not contain a square factor > 1

If $N = 8$, $W = 2^8 - 1 = 255$ which does not contain a square factor > 1

16. a) It is easy to find Pythagorean rectangles having integral sides and integral diagonals $(l, w, d) = (3, 4, 5)$ or $(5, 12, 13)$. But can you find a box (rectangular parallelepiped) having three distinct integers as dimensions, each of whose face diagonals is also an integer?

    b) If so, can you also find one whose body diagonal as well as face diagonals are integers?
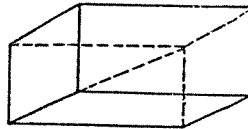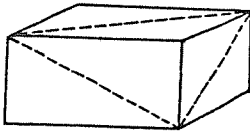
17. Use your TRS-80 to design some personal greeting or Christmas cards. Take photographs of the TRS-80 showing the design on the screen and enough of the keyboard to be recognizable. Send one to your authors.

PART II

You are ready to proceed on your own now. No one is expected to work all of the research problems in Lesson 5, but you should work several and at least read and consider the others. The way to learn computing is to compute, and then reflect.

The remainder of this book discusses useful techniques and additional BASIC instructions you may or may not need to know about. Lessons 6 to 17 may be read in any order desired, or ignored. Read about these techniques when you are ready to use them. You already have skill enough to solve many problems using the computer as a tool. We suggest you glance briefly at the summary below to ascertain the general content of the lessons beyond 5. Then read them when the need for such specialized information is felt. Read whatever interests you most, and continue with the micro-research problems presented in Lessons 5 and 17.

Lesson 6 is devoted to the use of the TAPE CASSETTE for storage of programs and data.

Lesson 7 is devoted to GAMES and TAPED PROGRAMS you may enjoy.

Lesson 8 discusses the EDIT instruction in greater detail.

Lesson 9 is an introduction to simulation, one of the most important uses of computers in today's world.

Lesson 10 discusses double precision variables (introduced in Lesson 5) before introducing subscripted variables and the fascinating (and vital) string variables (more details in Lesson 14). The lesson closes with arrays (matrices) and how to use them.

Lesson 11 is a collection of tips that may help you overcome or avoid difficulties. Remarks on saving memory space and computing time as well as a listing of the meanings of major error codes are presented in this lesson.

Lesson 12 Although the PRINT instructions already at your disposal are sufficient for most users, your TRS-80 Level II BASIC has a variety of other PRINT instructions and special commands that are frequently useful:

         PRINT TAB(   )   PRINT STRING$(K,"#")

         PRINT USING A$,K   PRINT CHR$(n)

Lesson 13 Lesson 4 may meet all your graphic needs, but Lesson 13 explains the pixel notation and introduces several time-saving techniques and instructions.

Lesson 14 extends your knowledge of string variables, and introduces elementary conversational programs in which the computer simulates conversation. The ROBOT COUNSELOR program and the CIPHER program may also interest you.

Lesson 15 explains how and where to look for additional information on computing with particular emphasis on the TRS-80 Level II BASIC. Several computer related journals are listed with brief annotations.

Lesson 16 discusses several BASIC instructions not used earlier in in this book.

Lesson 17 extends the collection of seventy-five microRESEARCH PROBLEMS. The way to learn computing is to compute and then to examine your programs and results to see if you could have devised a better program.

The discussions presented Lessons 5 and 17 are planned to help you develop efficient as well as effective programs and to whet your problem solving ability. Many of the problems presented are readily extended. Their solutions frequently suggest further explorations worthy of your talent. This is why they merit the adjective "RESEARCH". The wide variety of problems includes something for every taste: several arithmetic explorations, greeting card graphics, Haiku poetry, logical decisions, calendar problems, dart and target games, loan and finance programs, treasure hunt games, caricatures, speech timer, puzzles, recursive functions, polygonal maps, a medical emergency prompter, number theory problems, lattice problems, magic prime squares, graphs, monkeys at typewriters, amicable and sociable numbers, dance partners and the eight queens problem.

THE IMPORTANT THING IS TO HAVE FUN WITH YOUR micro COMPUTER

TAPE CASSETTE for programs and data storage.

Occasionally you have a special program worth saving for future use or to show off to a friend. Put it on tape. Tape is also handy for storing data.

Each TRS-80 comes with a tape recorder for storing programs and data that you wish to save. It uses regular (high quality) tape cassettes. I personally prefer the short 30 minute (15 minutes per side) tapes rather than the 45 minute, 60 minute or 90 minute tapes since the tape recorder starts, stops and operates more uniformly when the reel contains less tape. Try it and suit yourself. Radio Shack sells a special 10-minute tape for use on TRS-80 which has no "leader" on it---but any high quality 30 minute (15 minutes per side) tape will serve--just so you don't try to record on the leader.

For readers who have older CTR-40 and CTR-41 tape recorders:
The original tape recorders supplied with TRS-80s were CTR-41's on which the REM jack had to be unplugged before the tape was repositioned. The CTR-41 also requires the use of a "dummy plug" in the MIC jack at all times. The CTR-80 now furnished should <u>not</u> have a dummy plug inserted at any time.

Instead of unplugging the REM jack on the CTR-41 you can use a short program.

```
9000  OUT 255,4
9010  INPUT "REPOSITION TAPE, PRESS (ENTER) WHEN READY" ; A$
9020  END
```
Then type RUN 9000 when you wish to reposition the tape.
For readers with pre-1980 CTR-80 tape recorders:
The more recent CTR-80's can use "fast-forward" or "rewind" without unplugging the connection between TRS-80 and the cassette at the REM jack.

This is much nicer.

In either case the volume control was critical in getting usable programs and data back into the TRS-80 -- on early (pre-July, 1979) TRS-80s.

The following may be helpful:

Radio Shack announced (Spring, 1979) a "fix" that they will install free to make the recording level less sensitive on existing TRS-80's. Current TRS-80's already have it installed. Your TRS-80 Level II keyboard has a catalogue number on the underside of the case. If the catalogue number ends -1 (example 26-1006-1) the modification has already been made. If not, it will be worthwhile to take the keyboard unit to Radio Shack to have the free "fix" made. Cassette storage is much easier to use thereafter.

If your CTR-80 cassette player was manufactured before February, 1979 (see date stamped on cassette box—Feb., 1979 = 2A9) take the cassette player to your Radio Shack store, too. They will install a small capacitor in the innards of the electronics to avoid a possible "noise spike" on your recorded tape if you push the stop button while it is reading tape, an infrequent but annoying happening. If the date of manufacture is Feb., 1979 or later (say 6A9 or 1A0) this extra protection has already been installed at the factory. We recommend setting modified CTR-80 cassette recording level at 4 both for recording and playback. Once you find the best volume for your system, a drop of white correction fluid (Snopake, Liquid Paper, etc.) or paint will mark the location for future reference.

In any case, the usual tape care is essential.

1. Don't leave cassettes near a magnetic flux source like your power supply or near a magnet as in a motor or a speaker. Also, don't expose cassettes to extreme heat or cold.

2. Keep your cassette capstan and rollers clean. Dirty heads cause lots of troubles. If you don't know how to clean them, your nearby Radio Shack or tape recorder store will be glad to show you, and sell you a cleaning kit (<$2).

<u>To load a tape that already contains a program</u>

Plug in everything carefully.

110   volt (Don't expect to use the batteries.) black power cord

DIN   plug to TRS-80 keyboard hole labeled TAPE with

pip on plug in slot on socket ,

3 plugs on other end of cable connect to cassette player

BLACK plug into EAR hole

LARGE GREY plug into AUX hole

SMALL GREY plug into small MIC hole (CTR-80) or REM hole (CTR-41)

If you are using CTR-41 put dummy plug into MIC and leave it there.

If you are using CTR-80 do <u>not</u> use dummy plug--just leave big MIC hole empty.

NEXT:

Adjust the position counter back to zero. (They register differently on CTR-41 and CTR-80, so be a bit suspicious.)

Check the volume adjustment. (This is fussy, experiment a bit.)

CTR-41   5 to 8

CTR-80   3.5 to 4.5 (user-generated); 4 to 5 (commercial tape)

Push $\boxed{\text{PLAY}}$ key on cassette.

Type CLOAD or CLOAD "ℓ" and $\boxed{\text{ENTER}}$ on Keyboard (where ℓ is letter identifying your program).

The cassette should be turning now.

Soon the TRS-80 video screen will show two asterisks.

Level 1　　　　　　*＊　　　　　　　　　　　**　　　Level II
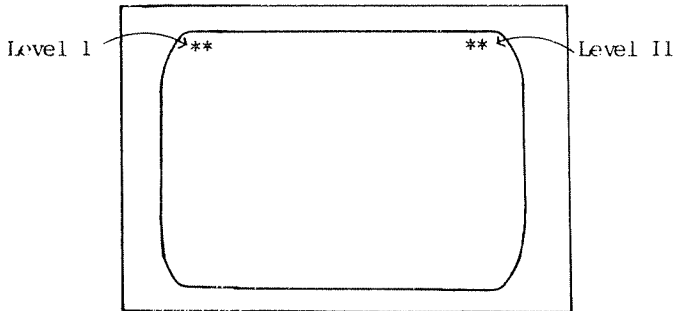
If all is going properly, one asterisk will blink irregularly.

When it finishes loading, the cassette will halt the tape and the display screen will show

READY

>_

Depress the [STOP] button on the cassette (even though the tape has halted). *Do not leave the* [PLAY] *key depressed when not in use.*

Type LIST [ENTER] .

If the program seems OK, type RUN [ENTER].

If no program is present (or if the asterisks appeared but did not wink) lower the volume slightly; rewind the tape and try again.

If something loaded, but the last part seems all jumble and gibberish, increase the volume slightly; rewind the tape and try again.

<u>To SAVE a program currently in the computer</u>

Set up the cassette recorder as described before; be sure plugs are set firmly in the proper holes.

Check the volume setting.

CTR 41   4–6
                         (Try mid-range first.)
CTR 80   3.5–4.5

Rewind the tape.(Unplug REM jack only if on CTR–41 or use program fix.)

Reset the tape counter to 000.

Advance the tape to get past the "leader" if you are using standard tape—or to a blank place if the tape already contains some programs or data. (I usually begin programs at 10 on the tape and leave between 10 and 19 "counts" of blank tape between programs. This may be overly cautious, but it sure makes it a lot easier to find things.)

Note the program name and where it begins on the index card with your cassette or on the cassette label.

Depress both RECORD and PLAY keys on the cassette recorder (surprise— yes both should be down to record--so the TRS–80 can control things properly).

Type CSAVE " " (with a letter to identify the program between the " ").

The tape should begin to advance and the TRS–80 will record your program on the tape.

When the program is recorded, the video screen will display as usual.

```
READY
>_
```

Depress the   STOP   button on your recorder. <u>This</u> <u>is</u> <u>important.</u>

Note where the program ended on your index card or the cassette label.

Rewind the tape to the beginning of the program (see index card and counter).

Depress   PLAY   only on cassette player.

Type CLOAD ? to check the tape against the program in the TRS–80 memory. If the tape checks properly, the screen will display READY, otherwise screen will display BAD and you should CSAVE " " it again.

I record each program twice, just in case anything happens. You'll have more space than you can use on a 15-minute per side tape. (Use both sides, of course.)

Sound and Music

Some TRS-80 programs include sound or music as part of their output. Higher quality musical reproduction is obtained by using a separate amplifier and speaker. If you are interested in medium fidelity sound rather than high fidelity music production, quite satisfactory results may be obtained using the amplifier present in the cassette player furnished with your TRS-80.

First get the program in your TRS-80 in the usual fashion. Then unplug the black plug from the ear hole in the cassette recorder (leaving the two grey plugs in place). Plug the earphone that came with the tape recorder into the ear hole (or better yet plug a speaker in there---you can get a suitable jackplug or converterplug from Radio Shack).

Now you need to fool the tape recorder. Remove any tape cassette from the recorder. Push in the little lever at the back left side of the opening where the cassette would go. While you are depressing it, depress both the [RECORD] and [PLAY] buttons on the front of the recorder. These should stay down when you release the lever. Now RUN your program.
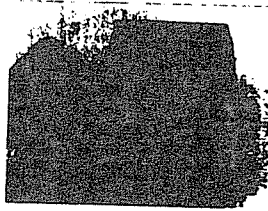
The following instructions are used to place data onto tape and to retrieve data from tape:

PRINT #-1,   PRINT #-1,X,Y,B$,A$

Output to cassette #1 from computer

INPUT #-1   INPUT #1,X,Z,B$

Input from cassette #1 to computer.

If you are having trouble with keyboard bounce (double letters from a single key stroke), use the debounce tape. More recent keyboards seem not to have as much trouble with this as did those Level II's delivered before 1980.

GAMES AND PLAYTOYS

Computers are magnificient game players. It isn't hard to <u>play</u> games which someone else has programmed – if they have done a good job of writing the program, but writing quality game–playing programs is difficult.

Here is a not very well written game. Put it on your computer; play it two or three times and then come back to the text and together we shall improve it.

```
  5  REM        TREASURE HUNT – FIRST TRY
 1Ø  CLS
 2Ø  X1 = RND(5Ø) : Y1 = RND(5Ø)
 3Ø  PRINT "A TREASURE IS BURIED IN A 50 FOOT BY 50 FOOT SQUARE.  IF YOU
     CAN LOCATE THE GRID POINT NEAREST IT, YOU WILL FIND THE TREASURE."
 4Ø  PRINT
 5Ø  INPUT "PLEASE TYPE COORDINATES  X, Y = ";X,Y
 6Ø  IF X = X1 AND Y = Y1 THEN 5ØØ
 7Ø  A = X – X1 : B = Y – Y1
 8Ø  D = SQR(A*A+B*B)
 9Ø  PRINT "SORRY YOU MISSED IT BY";D;"FEET."
15Ø  GOTO 4Ø
5ØØ  REM        TO HERE ONLY IF TREASURE IS FOUND
51Ø  FOR Q = 1 TO 8Ø
52Ø     PRINT @ RND(1ØØØ), "WHEEEE YOU GOT IT."
53Ø  NEXT Q
54Ø  GOTO 1Ø
```

Well – it's a game – and the program works if you typed it in correctly. But it isn't much fun. You really need more information than just the distance to the treasure.

It would help if the program told you whether X and Y are too big or too small. Do you want to try to revise the program on your own before you continue?

One of the most common games computer programmers play is "one-up-manship". Modifying existing programs is one form of the sport. An important way to gain programming skill is to adapt and improve a working program written by someone else. There is an axiom among computer programmers that says

```
Any program you have written,
   I can improve upon.
```

It has as its converse

```
Any program I have written,
   you can improve upon.
```

and together they have a corollary

```
Any program I have written,
   I can improve upon.
```

Each of these is probably true. In some cases the real question is whether or not it is worthwhile to improve on a program even if you can do so.

The answer depends upon how frequently the program will be run, and how much of your time will be required to improve the program.

Some ways it is frequently possible to improve programs are:

1. Make them use less computer time.
2. Provide output better suited to the user's needs.
3. Make the program easier to understand (and hence easier to modify and maintain).
4. Generalize the program so it is useful in solving other problems without sacrificing speed.

Programs designed to play games or solve puzzles somehow seem particularly subject to improvement, even by beginners. We shall look at several commercially available programs and how they can be improved. This is one of the most satisfying ways to improve your own ability as a programmer.

Let's go back and improve the treasure hunt program. The "heart" of the original program is

.

.

.

```
 5Ø   INPUT "PLEASE TYPE COORDINATES  X,Y =";X,Y
 6Ø   IF X = X1 AND Y = Y1 THEN 5ØØ
 7Ø   A = X - X1  :  B = Y - Y1
 8Ø   D = SQR(A*A+B*B)
 9Ø   PRINT "SORRY YOU MISSED IT BY";D;"FEET."
15Ø   GOTO 4Ø
5ØØ   REM  TO HERE ONLY IF TREASURE IS FOUND
```

.

.

.

```
54Ø   GOTO 1Ø
```

It would be nice to print another line of type right after 9Ø that tells us which way to go. Let's insert the following:

```
1ØØ   IF A > Ø   THEN A$="TOO BIG" ELSE A$="TOO SMALL"
11Ø   PRINT "YOUR X VALUE IS ";A$,
12Ø   IF B > Ø   THEN A$ = "TOO BIG" ELSE A$ = "TOO SMALL"
13Ø   PRINT "YOUR Y VALUE IS ";A$
```

Insert these instructions and run the program again. It is more fun this way, isn't it? Play it several times.

HEY – DID YOU NOTICE THAT THE PROGRAM CHEATS? YES IT DOES.

If you get either X or Y correct, but not both correct, it tells you to change them both.

Insert temporary debugging instruction

```
21  PRINT  X1, Y1
```

and run it again. The correct treasure location will now be printed at the top of the screen so you can test it. Try putting in two wrong values – Seems OK. Next put in the correct value for one coordinate and the wrong one for the other coordinate. Hmmm – The program still claims both values are either TOO BIG or TOO SMALL. That won't do, will it?

OK   OK   OK

We'll fix that too. Try

```
1ØØ   IF A > Ø   THEN A$ = "TOO BIG" ELSE IF A = Ø THEN A$ = "OK" ELSE
      A$ = "TOO SMALL"
12Ø   IF B > Ø   THEN A$= "TOO BIG" ELSE IF B = Ø THEN A$ = "OK" ELSE
      A$ = "TOO SMALL"
```

Try it – it works now. Remember to delete instruction 21 before you show it off to someone else.

There is really no reason we need always work on a 50 by 50 foot square. How about letting the player select the size of the square? We'll need an instruction to INPUT the size of the square.

15 INPUT "PLEASE TYPE LENGTH OF SIDE OF SEARCH SQUARE.";S

Remember to thwart the person who puts in 4.73 or –2 or some such joke.

16 S = INT(ABS(S))

Before continuing <u>you</u> should look through the program and see what other instructions need to be modified.

Instructions 2∅ and 3∅ need to be changed; our new program (still not perfect) is

```
 5   REM        TREASURE HUNT – SECOND TRY
1∅   CLS
15   INPUT "PLEASE TYPE LENGTH OF SIDE OF SEARCH SQUARE.";S
16   S = INT(ABS(S))
2∅   X1 = RND(S) : Y1 = RND(S)
3∅   PRINT "A TREASURE IS BURIED IN A";S;" BY ";S;"FOOT SQUARE.  IF YOU
     CAN LOCATE THE GRID POINT NEAREST IT, YOU WILL FIND THE TREASURE."
4∅   PRINT
5∅   INPUT "PLEASE TYPE COORDINATES  X,Y =";X,Y
6∅   IF X = X1 AND Y = Y1 THEN 5∅∅
7∅   A = X – X1 : B = Y – Y1
8∅   D = SQR(A*A+B*B)
9∅   PRINT "SORRY YOU MISSED IT BY";D;"FEET."
1∅∅  IF A > 0 THEN A$ = "TOO BIG" ELSE IF A = ∅ THEN A$ = "OK" ELSE
     A$ = "TOO SMALL"
11∅  PRINT "YOUR X VALUE IS ";A$
12∅  IF B > ∅ THEN A$ = "TOO BIG" ELSE IF B = ∅ THEN A$ = "OK" ELSE
     A$ = "TOO SMALL"
13∅  PRINT "YOUR Y VALUE IS ";A$
15∅  GOTO 4∅
5∅∅  REM    TO HERE ONLY IF TREASURE IS FOUND
51∅  FOR Q = 1 TO 8∅
52∅     PRINT @ RND (1∅∅∅),   "WHEEE  YOU GOT IT."
53∅  NEXT Q
54∅  GOTO 1∅
```

Play the game a few times and then, after correcting the spacing on instructions 3∅, 1∅∅ and 12∅ if needed, show it to a friend.

What happens if someone inserts .5 or ∅ as the value for S? Maybe we need another smart aleck guard at 17.

17 IF ABS(S) = Ø THEN S = 5ØØ : PRINT "ALL RIGHT, SMARTY, I'LL FIX YOU. TRY THIS."

Modifications of this treasure hunt program can provide the first approximation of a number of much more interesting games.

How about adding <u>Depth</u> at which the treasure is buried as well as X,Y locations?

Could you modify the coordinate game to play the child's game "BATTLESHIP" in which the computer selects positions for several targets which you must locate?

Can you insert graphics which will show you where your choices have been and the corresponding D value rather than telling you which way to move?

Try a few modifications of your own.

Here are some games programmed (or adapted) by secondary school students in Oklahoma in Spring, 1979. If you find unfamiliar instructions, see Lesson 16 or your TRS-80 LEVEL II BASIC Reference Manual.

```
  5  CLS
  7  PRINT "THIS IS A GUESSING PROGRAM.  YOU ARE TO GUESS A FRACTION N/D
     WITH 1<=N<=10, 1<=D<=10."
  9  PRINT
1ØØ  G = RND(1Ø)/RND(1Ø)
11Ø  PRINT "I HAVE A NEW FRACTION FOR YOU TO GUESS.  PLEASE TYPE YOUR
     GUESS IN DECIMAL FORM."
12Ø  INPUT X
13Ø  IF X = G THEN PRINT "WHEEE...YOU GOT IT!": GOTO 1ØØ
14Ø  IF X > G THEN 17Ø
15Ø  PRINT "YOUR GUESS IS TOO SMALL.  TRY AGAIN PLEASE."
16Ø  GOTO 12Ø
17Ø  PRINT "YOUR GUESS IS TOO BIG.  PLEASE TYPE IN ANOTHER GUESS."
18Ø  GOTO 12Ø
```

Play the game; then consider any desirable changes. One student suggested adding the following: *(However, it did not work as anticipated. Can you help?)*

```
  1Ø  ON ERROR GOTO 9ØØ
9ØØ  PRINT "PLEASE TYPE THE FRACTION USING DECIMAL FORM, NOT N/D FORM."
91Ø  GOTO 9
```

THIS IS A HANGMAN GAME ADAPTED BY

```
  3   REM     KAREN JEANNE HENRY
  4   REM     R1  BX 580 MCLOUD, OK 74851
  5   REM     (405) 273-1133
 10   REM     THE HANGMAN PROGRAM IS SELF EXPLAINED.  THE PERSON PLAYING
              WILL INPUT A LETTER AND THE COMPUTER WILL TELL YOU IF YOU
              ARE CORRECT OR NOT.
 20   REM     AS YOU INPUT EACH LETTER THE COMPUTER MAY OR MAY NOT CHANGE
              GRAPHICS
 25   REM     WHEN YOU FINISH THE GAME THE COMPUTER WILL VERIFY YOUR WIN OR
              LOSS BY CHANGING GRAPHICS AND/OR WORDS
 30   REM     X + Y ARE USED IN SET STATEMENTS IN FOR-NEXT LOOPS
 31   REM     D$( ) ARE THE WORDS, F$( ) ARE THE LETTERS OF THE WORDS
 32   REM     WD IS THE TOTAL NUMBER OF WORDS IN THE MAIN PROGRAM
 33   REM     W$ IS THE INPUTED LETTER L(N) + F1 ARE COUNTING DEVICES
 34   REM     K IS A VARIABLE OF HOW MANY WRONG LETTERS
 99   CLEAR 100
100   REM
102   CLS
103   WD = 10
104   K=0: FOR N=1 TO 8: L(N)=0: NEXT N
105   PRINT "**HANGMAN**"
106   FOR X = 9 TO 51 STEP 6
107   SET(X,15): SET(X + 1,15) : SET (X + 3,15) : SET(X + 2,15): NEXT X
108   FOR X = 70 TO 79: SET(X,44): NEXT X
109   FOR Y = 4 TO 44: SET(74,Y): SET(75,Y): NEXT Y
110   FOR X = 76 TO 101: SET(X,4): NEXT X
140   FOR Y = 5 TO 7: SET(101,Y): NEXT Y
150   IF FG = 1 THEN 160 ELSE D$(1) = "APPLE": D$(2) = "HEART": D$(3) =
      "STUDENT" : D$(4) = "TEACHER": D$(5) = "PENCIL" :D$(6) = "PAPER" :
      D$(7) = "PEN" : D$(8) = "WORK": D$(9) = "TIRED": D$(10) = "HARD"
160   DD = DD + 1: IF DD> WD THEN 2050 ELSE Z$ = D$(DD)
180   N = LEN(Z$)
190   F$(1) = MID$(Z$,1,1)
200   F$(2) = MID$(Z$,2,1)
210   F$(3) = MID$(Z$,3,1)
220   F$(4) = MID$(Z$,4,1)
230   F$(5) = MID$(Z$,5,1)
240   F$(6) = MID$(Z$,6,1)
250   F$(7) = MID$(Z$,7,1)
260   F$(8) = MID$(Z$,8,1)
270   IF F$(3)<>"" THEN 290
280   RESET (21,15) : RESET (22,15) : RESET (23,15) : RESET (24,15)
290   IF F$(4)<>"" THEN 310
300   RESET (27,15) : RESET (28,15) : RESET (29,15) : RESET (30,15)
310   IF F$(5)<>"" THEN 330
```

```
320   RESET (33,15) : RESET (34,15) : RESET (35,15) : RESET (36,15)
330   IF F$ (6) <> " " THEN 350
340   RESET (39,15) : RESET (40,15) : RESET (41,15) : RESET (42,15)
350   IF F$ (7) <> " " THEN 370
360   RESET (45,15) : RESET (46,15) : RESET (47,15) : RESET (48,15)
370   IF F$ (8) <> " " THEN 390
380   RESET (51,15) : RESET (52,15) : RESET (53,15) : RESET (54,15)
390   PRINT @ 388, "THIS WORD HAS "; N; "LETTERS",;
400   PRINT @ 452, "TYPE EACH LETTER" ;
410   PRINT @ 516, "PRESSING ENTER EACH TIME" ;
415   Z = 644
420   PRINT @ 896, " " ;
425   INPUT W$
427   PRINT @ 980, " " ;
430   FOR N = 1 TO 8
440   IF W$ = F$(N) THEN PRINT @ (258 +(3*N)), W$; : L(N) = 1 : FLAG = 1 :
      GOTO 450
450   NEXT N
460   IF FL = 0 THEN PRINT @ Z, W$: : Z = Z + 2
470   P = 0
480   FOR N = 1 TO 8 : IF L(N) = 1 THEN P = P + 1
500   NEXT N
510   IF P = LEN(Z$) THEN 2000
520   IF FL = 1 THEN FL = 0 : GOTO 420
530   ON K + 1 GOTO 540, 660, 710, 810, 910, 980
540   K = 1
550   FOR X = 95 TO 107 : SET (X,8): SET (X,16) : NEXT X
580   FOR Y = 10 TO 14 : SET (109,Y) : SET (93,Y) : NEXT Y
610   SET (94,9) : SET (108,9) : SET (108,15) : SET (94,15)
620   SET (97,10) : SET (105,10) : SET (101,11) : SET (101,12)
630   SET (95,12) : SET (96,13) : SET (97,14) : SET (98,14) : SET (99,14) :
      SET (100,14)
640   SET (101,14) : SET (102,14) : SET (103,14) : SET (104,14) :
      SET (105,14) : SET (106,13) : SET (107,12)
650   GOTO 420
660   K = 2
670   FOR Y = 17 TO 33 : SET (100,Y) : SET (101,Y) : SET (102,Y) : NEXT Y
700   GOTO 420
710   K = 3
720   FOR X = 103 TO 112 : SET (X,20) : NEXT X
750   Y = 19 : SET (117,14)
760   FOR X = 112 TO 116 : SET (X,Y) : SET (X+1,Y) : Y = Y-1 : NEXT X
800   GOTO 420
810   K = 4
820   FOR X = 90 TO 99 : SET (X,20) : NEXT X
850   Y = 26
860   FOR X = 85 TO 90 : SET (X,Y) : SET (X,Y-1) : Y = Y-1 : NEXT X
900   GOTO 420
```

```
91Ø   K = 5 : Y = 33
92Ø   FOR X = 1Ø3 TO 112 : SET (X,Y) : SET (X+1,Y) : Y = Y+1  : NEXT X
96Ø   SET (113,43) : SET (114,43) : SET (115,43) : SET (116,43) :
      SET (117,43)
97Ø   GOTO 42Ø
98Ø   Y = 42
99Ø   FOR X = 89 TO 98 : SET (X,Y) : SET (X+1,Y) : Y = Y-1 : NEXT X
1Ø3Ø  SET (85,43) : SET (86,43) : SET (87,43) : SET (88,43) : SET (89,43)
1Ø4Ø  RESET (1Ø1,12) : RESET (95,12) : RESET (96,13) : RESET (1Ø6,13) :
      RESET (1Ø7,12)
1Ø6Ø  FOR X = 97 TO 1Ø4 : RESET (X,14) : NEXT X
11ØØ  SET (97,14) : SET (98,13) : SET (99,13) : SET (1ØØ,13) :
      SET (1Ø1,13) : SET (1Ø2,13) : SET (1Ø3,13) : SET (1Ø4,13) :
      SET (1Ø5,14)
111Ø  PRINT @ 772, " AHHH! THAT'S TOO BAD"; : GOTO 2Ø1Ø
2ØØØ  PRINT @ 772, "CONGRATULATIONS! YOU WON!!!";
2Ø1Ø  FOR N = 1 TO 2ØØØ : NEXT N : CLS : GOTO 1ØØ
2Ø5Ø  REM     THIS CHANGES WORDS
2Ø55  ON  F+1   GOTO 2Ø6Ø, 2Ø7Ø
2Ø6Ø  D$(1) = "GEOMETRY" : D$(2)="ALGEBRA" : D$(3)= "TRIANGLE" :
      D$(4)= "FORMULA" : D$(5)= "SOLUTION" : D$(6)= "SUBTRACT" :
      D$(7)= "NEGATIVE" : D$(8)= "REDUCE" : D$(9)= "INVERSE" :
      D$(1Ø)= "FRACTION" : K=1 :  F=1 :  GOTO 170
2Ø65  DD = Ø : FG = 1 : GOTO 1ØØ
2Ø7Ø  REM     THE PERSON USING THIS PROGRAM CAN ADD MORE WORDS IN THIS SAME
      WAY, USING THE D$(SUBS), OR HE CAN USE DATA FILES AND STORE WORDS ON
      A TAPE RECORDER CONTROLLED BY THE COMPUTER.  USE   FOR N = 1 TO 1Ø :
      INPUT # -1 , D$(N) : NEXT N : GOTO 2Ø65
```

This program <u>will</u> fit on a 4-K Level II TRS-80, but you may need to "squeeze out" most of the spaces as suggested in Lesson 11.

```
1Ø6Ø  FORX=97TO1Ø4:RESET(X,14):NEXTX
```

is hard to read, but requires less computer memory. Lines 3 to 34 may be omitted, if necessary.

After it is up and working, you may wish to try to improve it by including additional words from a cassette file. See Lesson 6.

<u>R E V E R S E</u>

```
1Ø   REM      * REVERSE *        BOB YARBROUGH    4-78
2Ø   REM      RANDOM DIGITS ARE FLASHED.  OPERATOR MUST ENTER NUMBER
21   REM      FORMED BY WRITING DIGITS IN REVERSE ORDER
3Ø   REM      L IS INITIAL LENGTH OF STRING OF DIGITS
31   L=3
4Ø   REM      A IS LENGTH OF COUNTER
41   A=1ØØØ
5Ø   REM      G IS NUMBER OF CORRECT RESPONSES BEFORE STRING
51   REM      IS LENGTHENED
52   G=3
6Ø   REM      STRING IS SHORTENED WHENEVER AN ERROR IS MADE.
7Ø   CLS
8Ø   PRINT "YOU MUST ENTER DIGITS SHOWN IN REVERSE ORDER."
9Ø   PRINT
1ØØ  C=Ø
1Ø1  PRINT "NEXT STRING WILL BE ";L;" LONG."
1Ø2  Z=Ø
1Ø4  GOSUB 2ØØ
1Ø6  CLS
11Ø  FOR X=57 TO 68
112     SET(X,21)
114     SET(X,29)
116  NEXT X
12Ø  FOR Y=21 TO 29
122     SET(57,Y)
124     SET(68,Y)
126  NEXT Y
13Ø  FOR N=1 TO L
132     A(N)=RND(9)
134     Z=1Ø*Z + A(N)
136  NEXT N
14Ø  FOR N=L TO 1 STEP -1
142     PRINT @ 542,A(N);
144     GOSUB 2ØØ
145     SET(6Ø,25)
146     SET(61,25)
147  NEXT N
148  CLS
149  PRINT
15Ø  INPUT P
152  IF P <> Z THEN 18Ø
16Ø  PRINT "CORRECT"
162  C=C+1
164  IF C < G THEN 1Ø2
166  L=L+1
168  GOTO 1ØØ
```

R E V E R S E (Continued)

```
18Ø    PRINT "WRONG    ***";Z;"***"
183    GOSUB 2ØØ
186    L=L-1
19Ø    GOTO 1ØØ
2ØØ    FOR M=1 TO A
2Ø2    NEXT M
2Ø4    RETURN
21Ø    END
```

Note: If you wish the digits to be displayed for a shorter period of
       time, change line 41 to:

  41 A = 5ØØ or some smaller value.

ASTEROIDS

```
1   REM   *ASTEROIDS*   MICHAEL BRIGGS      1-79
2   CLS : PRINT " MANEUVER THE SHIP THROUGH THE ASTEROIDS BY USING THE
    UP-ARROW KEY TO GO UP, THE DOWN ARROW TO GO DOWN, AND THE < AND >
    KEYS FOR LEFT AND RIGHT."
3   Z = 15
4   DIMA(Z,15)
5   PRINT
6   INPUT "PRESS ENTER TO START" ; D
7   LL = 150
9   CLS
10  X = 63 : Y = 44
15  TI = 0
20  SET (X,Y) : SET (X,Y+1) : SET (X-1,Y+2) : SET (X+1,Y+2)
22  FOR G = 0 TO 127
24     SET (G.47)
26  NEXT
30  FOR L = 3 TO 15
40     FOR P = 1 TO Z
50        A(P,L) = RND(128)-1
60        SET(A(P,L),2*L)
70  NEXT : NEXT
80  PRINT @ 32,   "GO!"
90  FOR L = 3 TO 13
100    R = RND(3)+2
110    FOR P = 1 TO Z
120 S$ = INKEY$
122    TI = TI + 1
123    IF S$ = "" THEN 240
125    RESET (X,Y) : RESET (X,Y+1) : RESET (X-1,Y+2) : RESET (X+1,Y+2)
130    IF S$ = "+" THEN Y = Y-1 : GOTO 170
140    IF S$ = "↓" THEN Y = Y+1 : GOTO 170
150    IF S$ = "." THEN X = X+2 : GOTO 170
160    IF S$ = "," THEN X = X-2
170    IF X = 127 OR X = 1 GOTO 500
180    IF Y = 45 THEN GOSUB 1000
190    IF Y = 1 THEN 2000
200    IF POINT(X,Y) OR POINT(X,Y+1) OR POINT(X-1,Y+2) OR
       POINT(X+1,Y+2) THEN 1505
235    SET (X,Y) : SET (X,Y+1) : SET (X-1,Y+2)  : SET (X+1,Y+2)
240    RESET (A(P,L), 2*L)
250    A(P,L) = A(P,L)+R
260    IF A(P,L) >=128 THEN A(P,L) = A(P,L) -128
265    IF ABS(A(P,L)-X) < 2 THEN 900
268    SET(A(P,L),2 * L)
270    IF TI = LL THEN 2500
280 NEXT : NEXT
300 GOTO 90
```

Note: NEXT works as well as NEXT X, but the practice of omitting the variable can cause a problem in nested FOR...NEXT loops.

Note: The + won't show on the screen, but it is in there.

ASTEROIDS (Continued)

```
5ØØ  CLS
51Ø  PRINT  "YOU HAVE JUST RUN INTO A BLACK HOLE, SORRY!"
52Ø  GOTO 2Ø2Ø
9ØØ  IF 2*L    >= Y AND 2*L  <= Y+3 THEN 15Ø5

91Ø  GOTO 268
1ØØØ CLS
1Ø1Ø PRINT  " YOU HAVE CRASH-LANDED, BUT IT'S NOT TOO FAR TO THE BASE."
1Ø2Ø GOTO 2Ø2Ø
15Ø5 FOR BL= 1 TO 4Ø : RESET (X,Y) : RESET (X,Y+1) : RESET (X-1,Y+2) :
     RESET (X+1,Y+2) : SET (X,Y) : SET (X,Y+1) : SET (X-1,Y+2) :
     SET (X+1,Y+2) : NEXT : CLS
151Ø PRINT  "(AP) HOUSTON - NASA MISSION CONTROL REPORTS THE LOSS OF AN
     INTER-PLANETARY SHUTTLE IN ROUTE TO SATURN.  THE VESSEL APPARENTLY
     SUFFERED SERIOUS ASTEROID DAMAGE AND WAS UNABLE TO RETURN TO EARTH."
152Ø GOTO 2Ø2Ø
2ØØØ CLS
2Ø1Ø PRINT  "EXCELLENT NAVIGATION! CONGRATULATIONS ARE IN ORDER."
2Ø15 PRINT  "YOU STILL HAVE" ; (LL-TI)*1Ø ; "GALLONS OF FUEL LEFT."
2Ø2Ø PRINT : INPUT  "PRESS ENTER TO TRY AGAIN" ; D
2Ø3Ø GOTO 9
25ØØ CLS : PRINT "YOU HAVE RUN OUT OF FUEL.  YOU WILL CRASH-LAND IN
     APPROXIMATELY 15 MINUTES.  WE MIGHT EVEN SEND A SEARCH PARTY TO
     PICK UP THE PIECES." : GOTO 2Ø2Ø
```

Here is a brief program that clutters up the screen in an interesting
fashion. Try it. You might like it.

```
1ØØ POKE 16396,23
11Ø CLS
12Ø FOR N=14336 TO 1536Ø
13Ø    PRINT CHR$(PEEK(N)) ; "  ";     Don't overlook the final ; here.
14Ø NEXT N
15Ø GOTO 1ØØ
```

Then type RUN [ENTER] and hit several keys at once without using [ENTER] .
Try [Q] [X] [W] and then try various other keys with some or all of the
above.

←		S
7		S
W		C

To terminate the program, open the little door on the back left-hand side
of your keyboard unit and depress the button on the extreme left side.
You may need to turn your computer off and then back on again to assure
proper functioning of all keys.

You will find dozens of game programs in Softside, Computronics, Creative Computing, TRS-80 Computing (San Luis Rey, CA), PROG-80 (Milford, NH) and many other journals mentioned in Lesson 15. Some commercial games are rather expensive, costing $25 and more. Such games are still cheaper than ones you design, write and debug, if your time is valuable. Other tapes are available at a cost so low that you can hardly afford the time to type in the program and correct your typing errors. People's Software will send you a list of available program tapes (1980 price $7.50 for tape + 50¢ postage in U.S. for some tapes). They add more to the list all the time and will swap you free tapes for good original programs which you have written.

Tape 1 ($8) contains a whole collection of games, some trivial, some excellent. Other programs deal with business, banking, bio-rhythms, speed-reading, math refresher, etc. Tape 2 ($8) contains some slightly more sophisticated banking and business programs, math programs, statistical programs, etc. Tape 5 ($8) is more of the same. Tape 6 is People's Pascal, Version II, and the cost is $23.50. You may or may not be ready for the Pascal language.

You can learn a lot by ordering one of these tapes and then examining, adapting and improving the programs to suit your whim. BE SURE TO SPECI-FY WHETHER YOU HAVE LEVEL I OR LEVEL II BASIC. The address is:

People's Software
Computer Information Exchange
Box 158
San Luis Rey, CA 92066
Telephone: (714) 757-4849   (VISA or MASTERCHARGE accepted for phone
                              orders)

PRACTICE SESSION 7

1. Improve the treasure hunt program introduced in this lesson.

2. Either devise a number guessing game of your own or improve the game written by a secondary school student in this lesson.

3. Play Karen Henry's modified Hangman program and then modify it to use your cassette tape recorder to provide additional words.

4. Either write a game-playing program of your own or improve a game-playing program written by someone else. Almost any game-playing program can be modified and improved. Try out your own ideas.

5. See if you can determine where the "fuel supply" is stored in the ASTEROIDS program. Change the fuel allotment. Also, determine whether fuel is used up by "time elapsed" or by "number of keystrokes", and plan your playing strategy to take advantage of this knowledge.

6. Modify Bob's program to play REVERSE so the player can INPUT a value to determine the amount of time the numbers are displayed (the difficulty level).

7. Look in one of the journals listed in Lesson 15 and find a computer game. Put the game on your computer, debug and play it. Then, see if you can improve it.
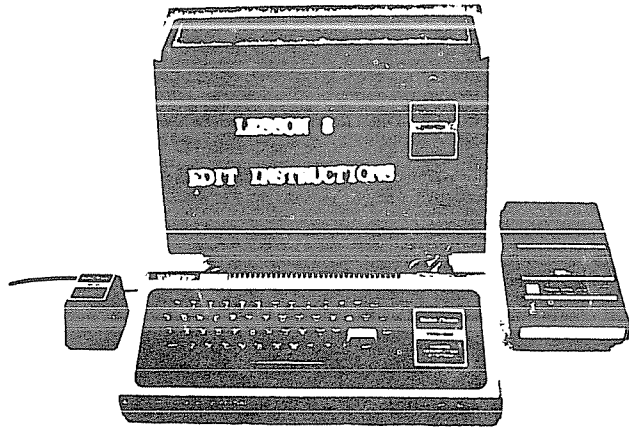
EDIT INSTRUCTIONS

The simplest way to change short lines in a program is certainly just to retype it, as we have been doing.

If you mistyped a program as

```
1ØØØ   FOR K = 1 BO   17        (error)
1Ø1Ø      PPRINT K;             (error)
1Ø2Ø   NEXT K
```

RUN   ⌷ENTER⌷

The computer will display

```
? SN ERROR IN 1ØØØ
READY
1ØØØ_
```

By depressing key ⌷L⌷ the computer will list line 1ØØØ and again display the line number, to facilitate correction.

```
?SN ERROR IN 1ØØØ
READY
1ØØØ FOR K = 1 BO 17
1ØØØ_
```

You may now either space over to just under the error using the space bar, or you may depress ⌷S⌷ and the ⌷B⌷ (the character in error). In either case, you will now have

```
?SN ERROR IN 1000
READY
1000 FOR K =1 BO 17
1000 FOR K =1 _
```

Next, depress [C] for "change" and depress [T] followed by [ENTER] to correct the line.

Type   LIST [ENTER]   to see that the change was made.

```
1000  FOR K = 1 TO 17
1010     PPRINT K;
1020  NEXT K
```

If you type   RUN [ENTER] you can go through a similar error detection and error correction routine for line 1010, but if you notice the "PP" when you LIST it, it is easier just to retype it as

```
1010  PRINT K;  [ENTER]
```

If you didn't note the error and type   RUN [ENTER]   , you will obtain

```
RUN
?SN ERROR IN 1010
READY
1010 _
```

Upon depressing [L]   the last line will show:

```
1010  PPRINT K;
1010 _
```

If you space the underline (cursor) over to directly under either of the letter P's

```
1010  PPRINT K;
1010  P_
```

and depress key [D]   , the computer will display:

```
1010  PPRINT K;
1010  P!P!
```

   The !! indicates the portion that will be deleted when you depress [ENTER] .

Depress   [ENTER]

Type LIST [ENTER]   and, sure enough, it shows:

```
1000   FOR K = 1 TO 17
1010      PRINT K;
1020   NEXT K
```

and RUN [ENTER] produces the expected result.

```
1 2   3  4  5  6  7  8  9  10  11  12  13  14  15  16  17
READY
—
```

You can always go into the EDIT mode to change a line by typing EDIT *line number*. For example, to change the 17 in line 1000 to a 37, one merely types:

EDIT 1000 [ENTER]

The computer responds

1000 _

Depress [L] , and you will have

```
EDIT 1000
1000 FOR K = 1 TO 17
1000 _
```

Note that the "1" of 17 is the second "1" in line 1000, so depress

[2] and then [S] and then [1]   (*Literally*, "*Seek the 2nd character 1*".)

The computer displays

```
EDIT 1000
1000 FOR K = 1 TO 17
1000 FOR K = 1 TO _
```

Now, depress [C] followed by [3] which gives

1000   FOR K = 1 TO 3_

Depressing [ENTER] will complete the line and enter the changes into the program, as   LIST [ENTER] will then show, and   RUN [ENTER] will demonstrate.

Chapter 9 in your TRS-80 Level II BASIC Reference Manual gives additional details, which are summarized here.

EDIT MODE SUBCOMMANDS AND FUNCTION KEYS

Subcommand/Function Key	Function
EDIT *line no.* [ENTER]	where *line no.* is a valid line number containing an instruction. This sends the computer to EDIT that line.
[ENTER]	End editing and return to Command Mode.
[SHIFT] [↑]	Escape from subcommand and remain in Edit Mode.
n [Space-Bar]	Move cursor *n* spaces to right.
n [←]	Move cursor *n* spaces to left.
[L]	List remainder of program line and return to beginning of line.
[X]	List remainder of program line, move cursor to end of line, and start Insert subcommand.
[I]	Insert the following sequence of characters at current cursor position; use [SHIFT] [↑] to get out of insert subcommand.
[A]	Cancel changes and return cursor to beginning of line.
[E]	End editing, save all changes and return to Command Mode. Same as [ENTER]
[Q]	End editing, cancel all changes made and return to Command Mode.
[H]	Delete remainder of line and insert following sequence of characters; use [SHIFT] [↑] to get out of this insert subcommand.
n[D]	Delete specified number of characters *n* beginning at current cursor position.
n[C]	Change (or replace) the specified number of characters *n* using the next *n* characters entered.
n[S]*c*	Move cursor to *n*th occurrence of character *c*, counting from current cursor position.

nKc   Delete all characters from current cursor
                position up to *n*th occurrence of character c ,
                counting from current cursor position.

    Let us try to use some of these EDIT instructions.   Type along on
your TRS-80 to help you follow what is taking place.

If you have the program line:

1ØØ   PLEASE PRINT THIS LINE.

and type

RUN [ENTER]

your screen will show

```
RUN
?SN ERROR IN 1ØØ
READY
1ØØ _
```

If you depress [L] the screen will show

```
RUN
?SN ERROR IN 1ØØ
READY
1ØØ   PLEASE PRINT THIS LINE.
1ØØ _
```

By now you realize that you need a PRINT instruction and that the message
needs to be in quotes if it is to be printed.

Depress [I] key   (Nothing seems to happen, but it has put the
                             editor into the insert mode.)

Type
PRINT "   (which will appear on the screen)

Depress [SHIFT] [↑]   (Again nothing seems to happen, but this
                             releases you from the insert mode.)

Depress [X]   (This moves to the end of the line and puts
                             the editor back into insert mode.)

Your TRS-80 will now show

```
?SN ERROR IN 100
READY
100 PLEASE PRINT THIS LINE
100 PRINT "PLEASE PRINT THIS LINE._
```

Note that the underscore at the end of the line shows you where the next
character will appear. It is right where you need to insert another quote.
To do so, type

" ENTER

Your correction is now complete.

If you type

RUN   ENTER

    the program will print the desired message.

Note: If, back at the beginning when you ran 100 PLEASE PRINT THIS LINE
and your screen showed

```
>RUN
?SN ERROR IN 100
READY
100 __
```

you forgot to give the EDITOR a command by depressing I (or some other
command key) and just started to type PRINT, an unexpected action would
have taken place.

The editor would ignore P R as unacceptable commands. It would then
use I as the "insert" command and would have inserted N T.

This is not at all what you wanted.

EDIT takes a bit of care, but it is well worth the effort.

LESSON 9
SIMULATION
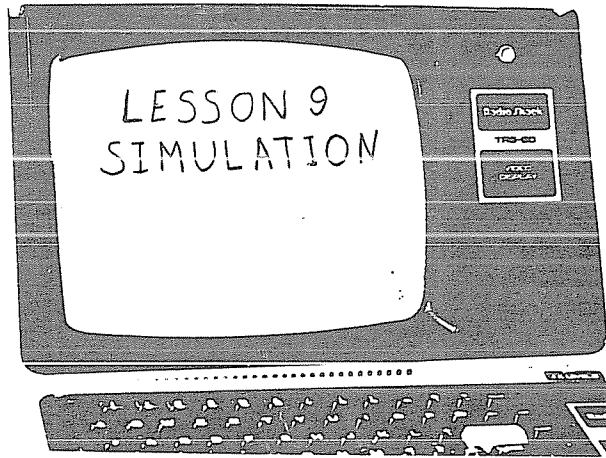
SIMULATION

A SIMULATION is a charade that uses a mathematical model to represent life-like experiments. The quality of the simulation is highly dependent upon how well the mathematical model represents the actual physical or social situation under study.

Many times it is difficult to compute a result of an actual experiment, either because the computation itself is too difficult to perform, or because the theory is not yet sufficiently developed. Even if the theory isn't well developed, it may still be feasible to use simulation to study the situation.

Simulation is one of the most important uses of modern computers. A simulation is a caricature. As in any caricature, it emphasizes the most prominent features under consideration and ignores the others.

We shall not attempt to simulate complex processes here. Instead, we shall simulate several rather simple situations — problems which you quite possibly could solve without resorting to simulation, but which will, nonetheless, illustrate the spirit of the game.

The heart of most simulation processes is some sort of random number generator.

The TRS-80 has an easy-to-use random number generator which you already investigated and used in Lesson 4.

RND($\emptyset$)   produces a random decimal between 0 and 1, not including either endpoint.

RND(K)   produces a random integer between 1 and K, including both endpoints.

RANDOM   is sometimes used once at the very beginning of programs that will be used frequently. It reseeds the random number generator so that the same sequence of RND ( ) values is not produced on each run.

Let's begin with a simple program to simulate five rolls of two dice.
Each die will produce one of the values 1, 2, 3, 4, 5, 6 at random. We
wish to display the value shown on each die as well as their sum. For
convenience, the individual dice are represented by R and G (for Red and
Green, to tell them apart).

```
100   REM    DICE THROWING PROGRAM
110   FOR K = 1 TO 5
120      R = RND(6)
130      G = RND(6)
140      S = R + G
150      PRINT "DICE SHOW" ; R ; G , "SUM =" ; S
190   NEXT K
```

Get this up and running before you go further.

Now imbed the above in a program that will collect statistics on how
frequently each sum occurs in 100 rolls.

```
10   DIM T(12)
20   REM        ZEROS T(2) TO T(12) FOR FUTURE RECORDS OF TOTALS
30   FOR S = 2 TO 12
40      T(S) = 0
50   NEXT S

100   REM    DICE THROWING PROGRAM
110   FOR K = 1 TO 100
120      R = RND(6)
130      G = RND(6)
140      S = R + G
150      PRINT "DICE SHOW" ; R ; G , "SUM =" ; S
160      T(S) = T(S) + 1  :  REM  THIS ACCUMULATES TOTALS IN T( )
190   NEXT K

300   REM        PRINTING OF TOTALS
310   PRINT
320   FOR S = 2 TO 12
330      PRINT S ; "WAS ROLLED" ; T(S) ; "TIMES IN 100 ROLLS OF TWO DICE."
340   NEXT S
350   PRINT
```

Once the program is running properly, you may change instruction 110 to
      110   FOR K = 1 TO 1000
to obtain a larger sample size. You will wish to change 100 to 1000 in
instruction 330 as well. Additional speed may be obtained by deleting
instruction 150 once you are sure the program is functioning as desired.

<u>A BIOLOGICAL SIMULATION</u>

Example 9-1

Suppose a biologist takes 100, equal-sized drops of liquid from a
liter flask containing bacteria which are randomly distributed (i.e.,
no tendency to cluster). Upon testing, it is discovered that 50 of the
samples are bacteria-free, while the other 50 contain 1 or more bacteria
per sample.* Assuming that the bacteria are distributed at random among
the samples, approximately what is the total number of bacteria in the
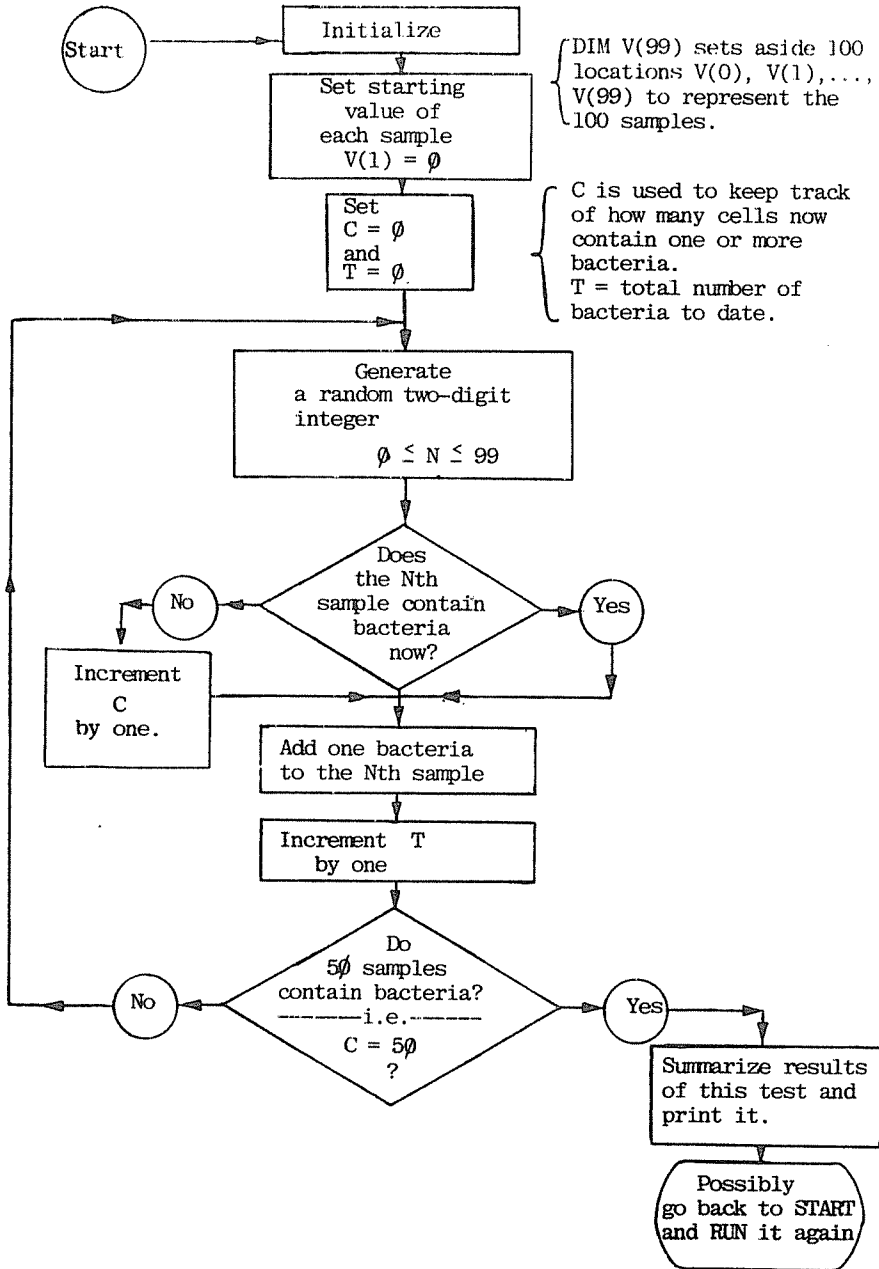100-drop sample?

The exact answer <u>cannot</u> be determined. However, it is possible to
obtain a statistical estimate of the desired number. You may or may not
have sufficient mathematical sophistication to solve this problem
directly, but it is an easy matter to simulate it.

Number 100 locations V(0) to V(99), and store zero in each. See
Lesson 10 for additional information on subscripted variables. Then
generate a sequence of random two-digit numbers. Each time the number N
is generated ($0 \leq N \leq 99$) we add one to location V(N). This is con-
tinued until exactly 50 of the locations contain non-zero numbers and 50
still contain zeros. At this stage, we have an estimate of the total
number of bacteria in the 100 samples, either by having kept track, or
by summing the V(N)'s.

Of course, the estimate may not be very good if we perform the
experiment only once. By running it several times, using different ran-
dom number sequences, we can obtain an average estimate that may be
reasonably close. The "proper" number of runs is a mathematical problem
of greater difficulty than we shall discuss here.

Do you see that this technique might be helpful? Actually, the
mathematics needed to justify and analyze techniques of this type is
highly sophisticated, but fortunately for many <u>users</u>, the techniques
themselves are comparatively simple to use. A <u>flow-chart</u> of a computer
program to simulate this experiment is given on the following page.

* The testing consists of adding a drop of reagent to the sample drop.
If it is bacteria-free, it will remain clear, but if the sample contains
one or more bacteria, it turns purple. However, the sample does not turn
twice as purple if two or more bacteria are present, so our test gives
<u>no idea of the total number of bacteria present</u>.

Start → Initialize

Set starting
value of
each sample
V(1) = 0

DIM V(99) sets aside 100
locations V(0), V(1),...,
V(99) to represent the
100 samples.

Set
C = 0
and
T = 0

C is used to keep track
of how many cells now
contain one or more
bacteria.
T = total number of
bacteria to date.

Generate
a random two-digit
integer

$0 \leq N \leq 99$

Does
the Nth
sample contain
bacteria
now?

No

Yes

Increment
C
by one.

Add one bacteria
to the Nth sample

Increment T
by one

Do
50 samples
contain bacteria?
———i.e.———
C = 50
?

No

Yes

Summarize results
of this test and
print it.

Possibly
go back to START
and RUN it again

```
100  DIM V(99)
110  FOR K = 0 TO 99
120     V(K) = 0
130  NEXT K
150  C = 0                        C=Number of Cells containing bacteria.
160  T = 0                        T=Total number of bacteria.

200  REM   GENERATE N: 0 <= N <= 99; INCREMENT V(N),
          TOTAL, AND IF APPROPRIATE COUNT.
210  N = RND(100)-1
220  IF V(N)  >  0  THEN 230
225  C = C+1
230  V(N) = V(N) + 1
240  T=T+1
250  IF C    <  50  THEN 210

300  PRINT "THE 100 DROPS CONTAIN A TOTAL OF" ;    T    ; "BACTERIA."
```

Before expanding this experiment, run it a few times. If it shows the same result each time, you may need to add the instruction

5   RANDOM   or   5 RANDOMIZE

to prevent the program from resetting the random number generator to the same place at the start of each run. After the program is debugged and running, you could imbed it in a FOR....NEXT loop to run it, say, 100 times and give information on the 100 trials. You might even run it 1000 or 5000 times and compute mean and standard deviation of the total number of bacteria to be expected in the 100-drop sample. You might also consider how to change the program if the number of contaminated drops found in the 100-drop sample were 35 or 70 instead of 50.

```
     For now merely add:
  90  G = 0 : REM   G  WILL BE USED TO ACCUMULATE THE GRAND TOTAL OR
                       CUMULATIVE TOTAL OF BACTERIA IN 100 RUNS

 105  FOR S = 1 TO 100
 300     PRINT "SAMPLE";S;"CONTAINS A TOTAL OF";   T   ;"BACTERIA."
 310     G = G + T
 320  NEXT S
 400  PRINT
 410  PRINT  " THE HUNDRED RUNS PRODUCED AN AVERAGE OF "; G/100 ; " TOTAL
               BACTERIA PER RUN "
```

In Practice Set 9 you will modify this program so that the number of runs can be specified by the user. You may wish to think a bit about which instruction will need to be changed. Actually, the simulated average of 100 runs should be expected to agree quite closely with the answer of 69 bacteria obtained using statistical theory.
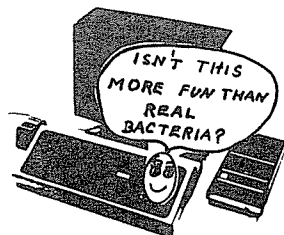
A SECOND BIOLOGICAL SIMULATION

EXAMPLE 9-2

Consider a superficially related, but actually different example. This time our biologist has a liquid which he knows contains exactly 200 amoebas (or fish) which again are distributed at random throughout the liquid. If he separates the liquid into 50 equal size samples V(1), V(2), ..., V(50), how many samples will be amoeba-free? (Also, how many will contain 1, 2, 3, 4, 5, 6, more than 6 amoebas?)

The computation on this is well within your understanding, but that is not really the purpose of this example. Let's simulate the problem by creating 50 storage cells, each of which initially contains zero amoebas. This time generate exactly 200 pseudo-random numbers N, with $1 \leq N \leq 50$. Each time the number N appears, add 1 to the number in location V(N).

```
1ØØ   DIM V(5Ø), M(8)              Sets aside space for samples V( )
11Ø   FOR K = 1 TO 5Ø             and final data collection M( ).
12Ø      V(K) = Ø                 This zeros 50 samples V(K).
13Ø   NEXT K

2ØØ   FOR B = 1 TO 2ØØ            This distributes the 200 amoebas
21Ø      N = RND(5Ø)              into the 50 samples.
22Ø      V(N) = V(N) + 1
23Ø   NEXT B

3ØØ   REM   ZERO THE M'S TO COLLECT DATA FOR FINAL REPORT
31Ø   FOR L = Ø TO 8
32Ø      M(L) = Ø
33Ø   NEXT L
```

(Continued on next page)

```
·100   REM   COLLECT DATA ON HOW MANY V-SAMPLES CONTAIN L AMOEBAS
·110   FOR B = 1 TO 50
·120      N = V(B)
·125      IF N < 7  THEN 430
·128      N = 8
430       M(N) = M(N) + 1
440    NEXT B

500    REM      PRINT OUT SUMMARY
510    FOR L = 0 TO 6
520       PRINT " THERE ARE " ; M(L) ; " SAMPLES WITH EXACTLY " ; L ;
          " AMOEBAS "
530    NEXT L
540    PRINT " THERE ARE " ; M(8) ; " SAMPLES CONTAINING 7 OR MORE AMOEBAS "
```

Once you have the program up and debugged and feel it is running
well, you may wish to include it in a FOR ... NEXT loop to run the
experiment several times and to tabulate the data in a more useful form.
This is left for you to do. Once again it will be discovered that the
average of, say 100, runs of such a simulation will agree quite closely
with the forecast obtained using statistical theory, in which the number
of amoeba-free samples is

$$50(1 - \frac{1}{200})^{200} \quad \text{which is approximately } 50*\frac{1}{e} \cong 18.4$$

The results of our simulation program correspond very closely to the
theoretical results.

A simulation of this type can also produce convincing evidence that
certain phenomena are not random. Let us assume that in your area P
persons died during D days of this year. If we assume that deaths occur
at random, then we are placing P markers into D cells just as we were in
the previous example, except · P = 200, and D = 50. In this case, the
agreement between the outcome of the simulation and actual data is not
good. Indeed, further analysis shows that deaths are not usually distrib-
uted at random, but rather that a high "successive-day-dependence" seems
to exist with alternate high and low death periods. This too can be
simulated, but the model is more sophisticated.

EXAMPLE 9-3

Now examine another simulation. This is essentially an Ehrenfest
model with three molecules (see your physics or chemistry teacher). For
simplicity it is posed as a "Guppy" problem.

Assume your aquarium contains 3 pregnant guppies. Because of their
gravid condition you prefer not to net the guppies, but instead to place
a "transfer cage" in the aquarium. At the start there are no guppies in
the transfer cage and three in the aquarium. . As time goes on the guppies
swim into or out of the transfer cage at random. When all three are in

the transfer cage, the door is closed and no further exchange is possible. The guppies are then transferred to a maternity aquarium. The number of guppies in the transfer cage at various times might be

$$0 \to 1 \to 2 \to 1 \to 2 \to 1 \to 0 \to 1 \to 2 \to 1 \to 2 \to 3 \quad \text{end.}$$

If you were unusually lucky, it might even be

$$0 \to 1 \to 2 \to 3 \quad \text{end.}$$

The problem is to determine the average number of <u>arrows</u> in the chain before all three fish are in the transfer cage.

Again there exist statistical techniques (for example Markov Chain techniques) which could be used to determine the desired result, but we prefer to simulate the process.

Before examining the solution proposed by your author, at least try your hand at flow charting the heart of this problem.

Here is <u>one</u> possible model. Use a random number generator to determine whether the next guppy goes into or out of the cage: If RND($\emptyset$) $\le$.5 put a fish into the transfer cage. If RND($\emptyset$) >.5 move out one, if possible. When all three fish are in the cage, stop the simulation.

```
1Ø  REM       FIRST GUPPY MODEL SIMULATION
2Ø  C = Ø : REM      C = # OF GUPPIES IN CAGE
3Ø  A = Ø : REM      A = # OF ARROWS
9Ø  PRINT "HISTORY=";
1ØØ R = RND (Ø)
11Ø IF R >.5 THEN 2ØØ
12Ø REM       HERE IF FISH ENTERS CAGE  R< = .5
13Ø A = A + 1
14Ø C = C + 1
15Ø PRINT C;
16Ø IF C = 3 THEN 5ØØ
18Ø GOTO 1ØØ
2ØØ REM       HERE IF FISH DOES NOT ENTER CAGE R> .5
21Ø IF C = Ø  THEN 1ØØ
22Ø A = A + 1
23Ø C = C - 1
24Ø PRINT C;
25Ø GOTO 1ØØ
5ØØ REM       HERE IF ALL 3 GUPPIES ARE IN CAGE
51Ø PRINT "SUCCESS AT LAST"
52Ø PRINT A; "ARROWS OR 'SWIM THROUGHS' TOOK PLACE"
53Ø PRINT
```

After debugging the above model you may enclose it in a program to run 25 times and give the <u>average</u> number of arrows for the 25 runs at the end of the history.

The preceding model may or may not represent this situation. Let us leave that for a chemist or physicist or guppy breeder to decide.

Before we show our model to an expert, it might be well to consider possible objections he could reasonably be expected to raise. Hmmm-- well, it does seem that the probability of a guppy leaving the cage should be greater when there are two guppies in the cage and only one outside than when the situation is reversed-doesn't it? Consider another possible model.

ANOTHER MODEL

You may easily discover a better simulation technique than the one suggested above. Be sure your next simulation takes into account that when the cage contains two guppies, then the next movement is twice as apt to be from the cage to the aquarium, as from the aquarium to the cage.

If you have trouble setting up your simulation model, here is an idea. Use three counters, one to represent each fish. If a given fish is in the cage, the corresponding counter will contain a 1, otherwise it will contain a zero. Two possible histories are:

(1)   Fish #1 → 0 → 1 → 1 → 0 → 1 → 1   end

   Fish #2 → 0 → 0 → 0 → 0 → 0 → 1   end

   Fish #3 → 0 → 0 → 1 → 1 → 1 → 1   end

which could be represented more compactly as

$$000 \rightarrow 100 \rightarrow 101 \rightarrow 001 \rightarrow 101 \rightarrow 111 \quad end.$$

Another possible history could be

(2)   Fish #1   0 → 0 → 1 → 1   end

   Fish #2   0 → 1 → 1 → 1   end

   Fish #3   0 → 0 → 0 → 1   end

which could be represented as

$$000 \rightarrow 010 \rightarrow 110 \rightarrow 111 \quad end.$$

Before you continue, write a program or flowchart to simulate this model.

In this simulation all three fish start outside of the transfer cage, i.e., F(1) = 0, F(2) = 0, F(3) = 0, THEN the computer generates a random number to determine which of the three guppies shall be the next one to change places (swim in if out, swim out if in cage). All three are in the cage when T = F(1) + F(2) + F(3) = 3.

The following flowchart summarizes the proposed program
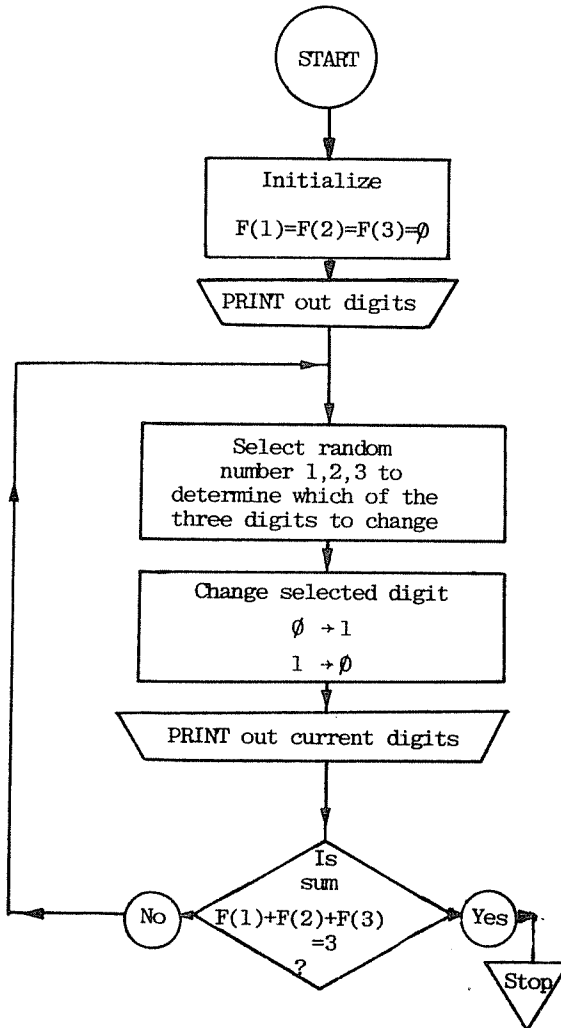
Here is one possible program for the previous flowchart -- but yours may
be just as good, or even better than this one, so don't change yours --
not yet, at least.

GUPPY -- SECOND MODEL

```
10    REM   GUPPY SECOND MODEL SIMULATION
20    DIM F(3)
50    REM      SET F1 = F2 = F3 = Ø  AS ALL GUPPIES ARE OUTSIDE CAGE
60    F(1) = Ø
70    F(2) = Ø
80    F(3) = Ø
90    PRINT F(1) ; F(2) ;  F(3) ;  " -> " ;
100   R = RND(Ø)
110   IF R > .667 THEN 300
120   IF R > .333 THEN 200
130   REM    HERE ONLY IF F1 IS TO MOVE
140   N = 1
150   GOTO 500
200   REM    HERE ONLY IF F2 IS TO MOVE
210   N = 2
230   GOTO 500
300   REM    HERE ONLY IF F3 IS TO MOVE
310   N = 3
500   REM    NOW READY TO MOVE FISH #N
510   IF F(N) = Ø THEN 550
520   IF F(N) = 1 THEN 600
530   PRINT "SOMETHING IS WRONG. F(" ; N ;") EQUALS"; F(N)
540   STOP
550   REM    MOVE F(N) INTO CAGE
560   F(N) = 1
570   IF F(1) + F(2) + F(3) <  3 THEN 90
580   PRINT F(1) ; F(2) ; F(3) ; "END--ALL IN CAGE NOW."
590   PRINT : PRINT
595   GOTO 60
600   REM    MOVE F(N) OUT OF CAGE
610   F(N) = Ø
620   GOTO 90
```

Try it out. Debug the program and when it is running smoothly include it
in a program to run the simulation 25 times and print out the average
number of arrows for the 25 runs, at the end of the histories.

PRACTICE SET 9

1. Simulate the roll of a pair of dice. After your simulation is up and
running, enclose it in an over-program that will roll the pair of dice
1000 times and keep track of how many times the sum shown is 2, 3, 4,...,
12. Compare the output of _your_ simulation with the theoretical outcome,
which you may either compute or discover in a book on probability, dice
or gambling.

2. Modify the program of Problem 1 so it assumes dishonest dice have been used in which the ⚁ is replaced with a second ⚃ .

3. Modify the program of Example 9-1, so the number of runs can be specified by the user.

4. Enclose the program of Example 9-2 in a FOR...NEXT loop that will permit you to run the experiment 500 times and tabulate a summary of the data obtained.

5. Explain the purpose of instructions 425 and 428 in the program of Example 9-2. Did the given program ever use M(7)? Why do you think the author left it so?

6. Create a flow chart for your final revised version of Example 9-2 (Problem 4) and use it to explain what your program does to a friend.

7. A batch of 80 ounces of plastic contains 100 small metallic impurities. Assume that the plastic is well mixed and that the impurities are distributed at random throughout the plastic when it is molded into 160 half-ounce replacement heart valves. If a valve contains one or more impurities it must be discarded. It is quite possible that anywhere from 60 to 159 metal-free valves may be obtained from the given batch. Write a computer program to simulate one hundred 80-ounce runs and determine the average number of metal-free heart valves you would expect on the basis of your simulation.

8. Extend the program of Problem 7 slightly so that in addition to giving you the average of the hundred runs, it will also keep track of the largest and the smallest number of metal-free valves that were manufactured from a single batch on the hundred runs.

9. Assume that in Shire approximately 100 Hobbits die in a given 30-day period. Write a computer simulation that will print out a table of twenty lines of output data, each line representing a 30-day period with 100 random deaths distributed among the 30 days. Print a reasonably clear header before the 20 lines of output data, since the Shire Health Department wishes to use your data for comparison with actual death rates.

10. A well-known and interesting problem asks the question, "In a set of thirty persons selected at random, what is the probability that at least two of the thirty persons have the same birthday (anniversary date) (i.e., same month and date of birth, but not necessarily born in the same year)?" How would you simulate this problem?

11. Write a program to simulate Problem 10 assuming 365 days per year. After debugging your central program, expand it so you run your simulation for 50 sets of 30 persons before you print out the approximate simulated experimental probability. Compare your simulated result with that computed. (See "the birthday problem" in a text on probability, if you need help.) Open Who's Who to some page and take the first 30 birthdays you find for a similar experiment.

12. Simulate some simple game of your own choosing and have the computer summarize the results. Suitable games might be "matching pennies", "put and take", or some very simple dice game but not anything as complicated as "Yahtzee" or "craps".

13. Expand your program for Example 9-1 or Example 9-2 to run 1000 times and compute standard deviation as well as mean of the numbers involved.

14. Debug the first guppy model. After it is debugged and running, include it in an "over-program" that will run the model 25 times; printing a history each time and give the average number of arrows per run at the end of 25 runs. If your runs all produce the same history, there is apt to be something wrong with the way you invoked the random number generator. Fix it up and rerun the program.

15. Debug the second guppy model. After it is debugged (and checked that it doesn't always give the same history) incorporate your debugged program into a program that will run the model 25 times, print a history each time and give the average number of arrows per run. Compare the results of model 1 and model 2. Which do you feel is a better representation of the problem? Why?

16. Write a program to simulate a penny toss between two persons if the first person starts with P1 pennies and the second with P2 pennies, where P1 and P2 are input for the program. Your program should print out the number of pennies each person has after each toss and stop as soon as either person has lost all his pennies.

17. Write a program to simulate a ping-pong game with the initial input being two numbers (P1 and P2) which represent the probability of player 1 and player 2 returning a given shot successfully. Change your model a bit to make it more realistic if you can.

18. Same as Problem 17, but for the game of tennis.

19. If you were to write a simulation of a baseball game, what variables would you wish to be able to read in and thus vary from run to run? Discuss a possible simulation with a colleague. Make a rough flow chart of your proposed simulation.

20. The dice game known as "craps" is played by noting the total number of points, when two dice are rolled, as follows:

    1. If, on the first roll, you roll 7 or 11, you win.
    2. If, on the first roll, you roll 2, 3, or 12, you lose.
    3. If, on the first roll, you roll a 4, 5, 6, 8, 9, or 10, this number becomes your "point" and you continue rolling the dice until you either roll your "point" again or you roll a 7. If you roll your point again before you roll a 7, you win. If a total of 7 turns up first, you lose.

Use the computer to simulate a craps game. Have the output show, on one line, the actual history as well as the first win or loss of the game.

21. Rearrange your program of Problem 20 so that you can simulate the game of "craps" under the assumption that each die carries the spots 1-5-3-4-5-6, instead of the usual arrangement.

22. Write a program to simulate the deal of 52 playing cards with NC cards per player and NP players. Your program should read in the values for NC and NP and check that NC*NP ≤ 52 before starting to deal. One way to deal using RANDU is to assume there are 52 cards numbered from 1 to 52 inclusive. You may wish to use DIMENSION C(52) and set the value of each C(I)=0 at the start, then once that "card I" is dealt, change the value of C(I) to +1 so that you do not deal the same card twice. (This causes hard feelings!) You may wish to store the hands in an array DEAL(NC,NP) by storing the numbers I, but in printing out the results, you should translate the stored integers into the usual suit and value designation. Use your program to deal 5-card poker hands to 8 players and also to deal 13-card bridge hands to four players.

LESSON 10

TYPES OF
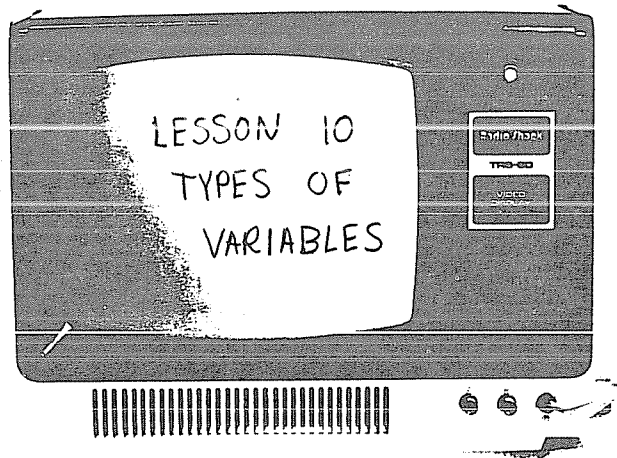
VARIABLES

VARIABLE TYPES

BASIC uses four distinct data types for its variables. Each variable may be forced to be any of the four types, either by using DEF statements or by the use of special type designators described below. The four types are:

Single Precision Floating-point (6 digits of accuracy)
Double Precision Floating-point (16 digits of accuracy)
Integer (between -32768 and +32767)
String (up to 255 alphabetic or numeric characters)

The first three types are used for numerical computation and storage. String variables are never used in arithmetic. They permit the storage, examination and comparison of sequences of letters, numbers, blanks and special characters.

a) **Single Precision Floating-point Variables** **DEFSNG A,B,W-Z**

Single precision floating-point numbers are the type we have been using for most of our computation. It is the option which the TRS-80 automatically elects for you unless you instruct it to do differently. Such computation is carried out to (the binary equivalent of) 7 decimal digits of precision inside of the computer, but <u>only 6 digits of precision are printed</u> on the CRT screen.

Since single precision is the automatic default condition, it is <u>not</u> necessary to use DEFSNG unless you need to change the precision of variables previously defined as INTEGER or DOUBLE PRECISION.

b) **Double Precision Variables**

You are already aware (See Lesson 5.) that it is possible to

obtain variables of greater accuracy (16 digits instead of 6 digits) for a variable by inserting

  DEFDBL X,Y

near the beginning of the program. This causes <u>every variable</u> beginning with the letter X or the letter Y to be double precision (17 digits internally, 16 digits displayed). It is also possible to indicate a range of initial letters in a DEFDBL statement. For example, DEFDBL A,C,S-W will cause every variable beginning with any of the variables A,C,S,T,U,V, W to be double precision unless it carries a special designator such as ! or % (see below).

c) <u>Integer Variables</u>

  It is also possible to specify that some variables are of <u>integer</u> form rather than the usual floating-point numbers. On short programs it is not worth the effort to do so, but long-running programs frequently can be speeded up noticeably by inserting

  DEFINT K,L,B-E

early in the program for those variables you know will always be integers in the range from -32768 to +32767. The DEFINT statement given above forces every variable beginning with K,L,B,C,D,E to be integer form. One of the most important savings is to make integers of the variables used in FOR...NEXT loops when appropriate. (Obviously, they must not take on fractional values in the step size.)

d) <u>String Variables</u>

  It is also possible on your TRS-80 LEVEL II BASIC to define a <u>string variable</u> (indicated by annexing a $ to any variable name) used to store strings (sequences) of letters and other characters up to 255 characters per string.

  Examples A$, C$, GR$, P1$, P2$

You may also define regular variables as string variables by using DEFSTR but be careful in doing this because if you designate DEFSTR A,D-G then every variable beginning with A,D,E,F,G will be a string variable and as such may <u>not</u> be used in arithmetic. Your authors usually prefer to use the annexed $ to indicate a string variable.

Try this little program:

```
     NEW  [ENTER]
 10  CLS
100  INPUT "PLEASE TYPE YOUR NAME" ; N$
110  PRINT : PRINT : PRINT : PRINT
```

12Ø PRINT "HELLO", N$; "." : PRINT "I HOPE YOU ARE ENJOYING THIS COURSE."
13Ø GOTO 11Ø

   RUN   `ENTER`

Try it out on your computer. Depress `BREAK` to stop the program.

     The output is a bit fast to read isn't it? To slow down the program, make it count from 1 to 500 after it prints by inserting

125   FOR Q = 1 TO 5ØØ : NEXT Q

RUN it again.

Try several different versions of your name.
Try inserting your social security number or phone number instead of your name.
<u>String variables</u> and <u>string instruction</u> are discussed further in Lesson 14.

e) <u>Individual Variable Type Designators</u>

     It is also possible to make particular variables into integer variables by annexing a % sign after it — without changing other variables that begin with the same letter. For example

1ØØ   FOR X% = 1 TO 5Ø
11Ø   FOR K% = 1 TO 1ØØ
12Ø   PRINT K%;
13Ø   NEXT K%
14Ø NEXT X%

will run faster than

1ØØ   FOR X = 1 TO 5Ø
11Ø   FOR K = 1 TO 1ØØ
12Ø   PRINT K;
13Ø   NEXT K
14Ø NEXT X

The first program runs in about half the time of the second program

     It is possible to have particular variables as regular single precision floating-point variables (6 digit precision) by annexing a ! to double precison floating-point variables (16 digit precision). Likewise, it is possible to have particular variables as double precision floating-point variables by annexing a # to any standard variable name. The type designator (% = integer, ! = single precision, # = double precision) takes precedence over an earlier DEF statement.

Variable Type	Annexed Character	Examples	Typical Variables To Be Stored
Integer Variable	%	A%,X%,K%,NI%	-2746,-5,0,1,17,3000
Single precision (6 digit) variable	!	A!,BX!,XI!	4,-50.73,.1276,1.23456E+09
Double precision (16 digit) variable	#	A#,XI#,C#,BV#	1.234567887654321D+15

Note: If you wish to enter or use double precision constants, you can save a lot of trouble by placing a D at the end of it. Thus A# = 1234.7D. Also, use D in place of E to indicate the exponent of 10. Thus:

    A = 6.02486E+23
    A# = 6.024859316852145D+23

Note that B, B%, B!, B# and B$ may all be used in the same program and each is a name for a different variable.

SUBSCRIPTED VARIABLES

The statement

DIM X(15), Y(15)

permits you to have sixteen different X-values

X(0), X(1), X(2), X(3), X(4),..., X(14), X(15)

and sixteen different (associated?) Y-values

Y(0), Y(1), Y(2),...,Y(14), Y(15)

in the computer at the same time .

Note that
X(3) and X3 are different variables, and each is different from X.

Such "subscripted variables" are important in statistics, where it is common to have sets of related data, say 25 temperatures and the 25 related product yields. The following program permits you to input 25 sets (T,Y) of temperature-yield data.

```
10      DIM T(25), Y(25)
100     FOR K = 1 TO 25
110        INPUT T(K), Y(K)
150     NEXT K
```

Note that we have not used location T(0) nor Y(0), but we could have.

The program might be expanded to also compute the average T value and the average Y value by accumulating the sum of the T's and the sum of the Y's. After the sums are computed, divide each by the number of data pairs (here 25) and store the result in $T(\emptyset)$ and $Y(\emptyset)$

```
1∅    DIM T(25), Y(25)
9∅    T = ∅ : Y = ∅
1∅∅   FOR K = 1 TO 25
11∅      INPUT T(K), Y(K)
12∅      T = T + T(K)
13∅      Y = Y + Y(K)
15∅   NEXT K
16∅   T(∅) = T/25
17∅   Y(∅) = Y/25
```

(Note: This is only a fragment of a program. It permits you to enter data into T( ) and Y( ), but does not produce or PRINT anything.)

Note that T and Y were used _inside_ the FOR...NEXT loop rather than using $T(\emptyset)$ and $Y(\emptyset)$. This saves computer time. Most expert programmers consider it gauche to use a subscripted variable whose subscript does not change inside of a FOR...NEXT loop.

If desired, the program could be extended to compute standard deviation, correlation, mode, median, and other statistical measures. If you are familiar with statistics, you can extend the program yourself. You can even compute a "line of regression" or a "curve of regression" for the given data.

Our purpose was to introduce subscripted variables, a very handy tool for solving many problems.

ARRAYS

It is also possible, in Level II BASIC to store _arrays_ (matrices) of numbers (or letters or symbols). Consider a set of class grades for 20 students on each of six tests. Although it is possible to store student names, it is simpler to use student numbers instead.

The 20 by 7 array is:

	Col 0	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6
	Student ID no.	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6
Row 1	12516	86	74	92	71	77	64
Row 2	12617	57	62	71	69	79	85
Row 3	77969	94	98	.	.	.	.

147

Row 4	38491	.	.	.	.	.	.
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
Row 20	21201	87	93	91	.	.	92

We represent it in the computer by

DIM G(2Ø,6)

This gives an array with 21 rows and 7 columns, for storage. We shall
not use row 0 for the storage data in this problem, but shall use column
0 to store the student's ID Number.

The 94 grade on test 1 of student number 3(ID = 77969) is located in po-
sition (3,1). Position (2,4) contains the grade of 69 for the 2nd student
(ID = 12617) on the 4th test.

The following program will read in the grades for the C-th test, to be
supplied by the teacher. For C = 0, the student ID# may be typed in, if
desired.

```
1     REM     PROGRAM TO INPUT QUIZ GRADES FOR CLASS OF 20 STUDENTS
1Ø    DIM G(2Ø,6)
1ØØ   INPUT "TYPE TEST NUMBER PLEASE"; C
11Ø   FOR R = 1 TO 2Ø
12Ø      PRINT "TYPE GRADE ON TEST" ;C; "FOR STUDENT #"; G(R,Ø),
13Ø      INPUT G(R,C)
14Ø   NEXT R
```

The following program will display the entire array of scores, as
well as computing and printing the average grade on the six tests for all
20 students. It assumes the data is already stored in the computer(from
tape, or by hand, or ??). This program may be combined with that above.
If no grades are entered, the computer will print zeros.

```
2     REM     PROGRAM TO DISPLAY AND AVERAGE STUDENT GRADES ALREADY IN THE
              COMPUTER
1Ø    DIM G(2Ø,6)
2ØØ   FOR R = 1 TO 2Ø
21Ø      T = Ø
22Ø      PRINT "STUDENT #" ;R, "I.D.#" ; G(R,Ø),
23Ø      FOR C = 1 TO 6
24Ø         PRINT G(R,C);
25Ø         T = T + G(R,C)
26Ø      NEXT C
27Ø      PRINT "->"; T/6
28Ø   NEXT R
```

Examine the program to understand how it works. If you have any questions, ask your instructor.

Note the use of indented instruction inside each FOR...NEXT loop. This is not important as far as the computer is concerned, but is a very helpful practice to help humans to perceive the structure of your programs. Your author sincerely recommends the practice.

LESSON 11
TIPS AND
ERROR
MESSAGES
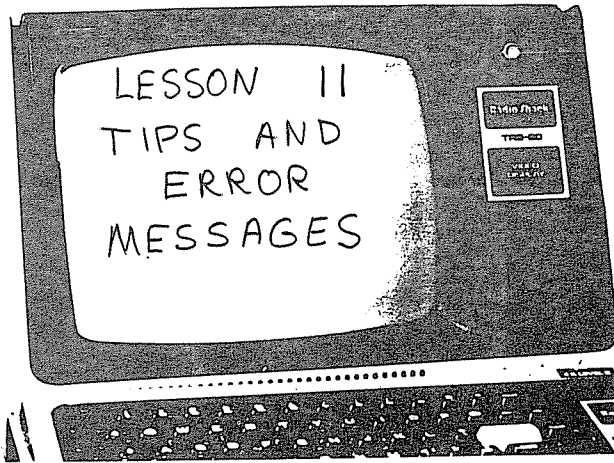
We have collected a few tips and cautions that you may never need, but that you still may find helpful.

TIPS

Don't let anyone spill beverage on or near your TRS-80. The only way to do this may be to forbid beverages near the computer. It is up to you, but a spilled Coke portends real grief forever afterward.

If you are plagued by double letters in your typing, it is because dust and dirt are forcing your TRS-80 to get multiple signals on a single key stroke. LEVEL II BASIC will have this problem more frequently than LEVEL I, since LEVEL II stores key strokes internally (great for fast typists). Keyboards manufactured in the 1980's seem to be less plagued with keyboard bounce than earlier TRS-80's. It is possible to remove the key tops and gently clean the contacts, but it is a lot easier to use the KEYBOARD DEBOUNCE (KBFIX) program furnished with LEVEL II TRS-80. It is a good idea to use a dust cover on the keybroad providing it is turned off (else heat may build up).

Begin your programs in line 1ØØ. Use lines 1 to 99 to identify your-self and program rather completely, as well as giving credit to any source you used before modifying your program.

If you would like your REM to stand out even more than it does by inserting 5 blanks after the REM, try depressing [↓] after typing REM and then type six or eight spaces before your REMark.

ERROR OS (out of string space) can frequently be remedied by insert-ing CLEAR n with a larger n value (say 1ØØ, 5ØØ, or MEM/4). The default value is CLEAR 5Ø whenever you turn on the TRS-80 or type NEW [ENTER].

If you use a BASIC program from a book or magazine, watch for RND(1) statements. Some BASIC's use RND(1) where the TRS-80 uses RND(Ø), namely, to generate a random decimal between Ø and 1.

If you type LLIST instead of LIST and do not have a line printer turned on, the TRS-80 will lock up. Open the small door at the back left of the keyboard unit and push the small round reset button.

The TRS-80 is designed for most room temperatures, but if you have clutter around it, that prevents air circulation. If you must use it in a hot place, put a couple of 1x1x5-inch blocks under the feet to permit extra air circulation.

Programs that produce a full screen of output can be ended with a tight loop (such as 9999 GOTO 9999) to prevent having break into the screen. Use BREAK   to get out of a tight loop.

```
READY
>_
```

If your program seems allright, but you get an SN ERROR that you can't identify, you may have depressed the SHIFT key while typing that line. (I tend to do this on @ .) Retype the line in question, watching your SHIFT key carefully.

If you are an electronics buff, by all means get the TRS-80 Micro-computer Technical Reference Handbook (1980, $9.95, Radio Shack). It has wiring diagrams you'll want to have. Remember, if you open the TRS-80 case, your guarantee is void.

Use CLOAD? to verify your Level II BASIC programs.

Double precision variables are not acceptable in FOR...NEXT loops.

If you have experienced problems with double precision variables not converting correctly (TRS-80 does not use decimal notation internally), it may be because you have not entered them correctly. If a double pre-cision variable has fewer than 7 digits when typed, follow it with the letter D.

Run this to see what is meant.

```
100   S = .1
110   D# = .1
120   PRINT S,D#
```

Your output will look something like

        .1   .1000000014901161

If you change 110 to

```
110   D# = .1D
```

all will be well. Try it and see.

Static electricity can play havoc with your TRS-80 and cassettes. If low humidity and carpet friction produce static electricity shocks in your computer room, do something about it. Remove the carpets or put down a non-static-producing rubber or plastic pad and increase the humidity in the room, if possible.

Don't force the 110V-plug on your video display into a socket or extension cord that is not designed to preserve polarity. One blade on the video 110V-plug is larger than the other--in this way, the video display chasis is grounded through the power cord. If you use an extension cord or multiple-outlet plug that does not preserve this polarity, you can get into trouble.

No doubt you have already discovered the HORIZONTAL HOLD and VERTICAL HOLD controls located on the rear of your video display. Once set, it should not be necessary to readjust them very often.

The BRIGHTNESS (B) and CONTRAST (C) controls are located on the front bottom right of your video display. Keep each in about the middle of the range. To lengthen tube life, DO NOT ADJUST TO MAXIMUM BRIGHTNESS OR CONTRAST. It may burn your tube if the display remains fixed.

Turn OFF the video display whenever you leave the computer, even if you leave the computer running.

Electrical interference from an elevator or other motor, a faulty flourescent light or even the switch in a typewriter are not apt to interfere with your TRS-80 unless you have added disk to your system. If you are considering disk or other expansion interface items, you should add a couple of line filters at your power source as well.

SAVING MEMORY SPACE

If you have only 4K of memory, you may find that unwelcome OM ERROR message signifying all of the available memory has been used or reserved. There are several things you can do to conserve memory space.

Cut down on the size of any arrays used.

Use multiple statements per program line with a colon between statements.

Use integer variables whenever possible—in FOR K% = 3 TO 3$\emptyset\emptyset$, for example. Use the same variable in all FOR...NEXT loops that are not nested.

Delete unnecessary parentheses in arithmetic statements.

Cut down on both the number and length of string variables.

Delete unnecessary spaces in all statements.

Eliminate extra variables, reuse variables no longer needed, keep variable names short.

If necessary, also delete REM statements.

ERROR OM can sometimes be cured providing you are using no string variables by inserting CLEAR $\emptyset$ at the beginning of your program.

OS ERROR frequently seems to be the result of a missing CLEAR n statement, where n is greater than 5$\emptyset$. Try CLEAR 2$\emptyset\emptyset$ or or CLEAR MEM/4.

SPEEDING UP PROGRAMS

Although computers are fast, they do run into time difficulties on long programs. Usually the solution is to think hard about the algorithm being used. It is frequently possible to devise a much faster algorithm once the problem is fully understood. There are also programming techniques, known to most experienced computists, that save considerable time.

Don't use subscripted variables inside of a FOR...NEXT loop unless the subscript is changing.

Combine several statements per line (separated by colons).

Use integer variables in FOR...NEXT loops that have integer step sizes.

Define the most commonly used variables first--this puts them near the top of the variable table and saves look-up time if many variables are involved.

Don't recompute values needed repeatedly; store them instead. (This is particularly vital inside of nested FOR...NEXT loops.)

Use small data sets and short FOR...NEXT loops in debugging long programs.

Use faster forms whenever possible.
B + B is faster than 2 * B.
.2 * C is faster than C/5. (Avoid division when possible.)
D * D * D * D is much faster than D ↑ 4.

Don't recompute trigonometric or log functions for the same value.
To evaluate $Y = 5 \sin^4 X + 8 \sin^3 X - 2 \sin^2 X$

use S = SIN(X)

Y = ((5 * S + 8) * S - 2) * S * S.

Use POKE graphics. This can cut graphics time to 1/5 of SET(X,Y).

Use stored constants (a letter like B or P) rather than numeric constants in the statements that are executed frequently.

Probably the most important thing you can do to make programs run faster is to analyze which subpart of your program uses the greatest amount of time and then rewrite that portion as a subroutine in Z-80 machine language. Call the subroutine using the USR(X) routine. However, that requires learning another language. See Chapter 8 of your TRS-80 Level II BASIC Reference Manual under USR(X) and a text on Z-80 machine language. Radio Shack has a book, TRS-80 Assembly Language Programming by Wm. Barden, Jr., (Cat. #62-2006) which will be helpful if you are sufficiently interested to wish to learn Z-80 chip language.

ERROR MESSAGES

We have discussed, under EDITING, Lesson 8, the appearance on the screen
of the message

?SN ERROR IN 1000

LEVEL II BASIC also has other messages that help you discover errors in
a program. The following ERROR messages are among those you may encoun-
ter. (If you have a disk on your system—consult the appropriate refer-
ence manual for additional messages.)

EXPLANATION OF ERROR MESSAGES

BS Subscript out of Range: An attempt was made to assign a subscripted
    variable or a matrix element with a subscript beyond the DIMensioned
    range.

CN Can't Continue: A CONT was ENTERed at a point where no continuable
    program exists. e.g., after program was ENDed or EDITed.

DD Redimensioned Array: An attempt was made to DIMension a matrix which
    had previously been dimensioned by DIM or by default statements. It
    is a good idea to put all dimension statements at the beginning of
    a program.

FC Illegal Function Call: An attempt was made to execute an operation
    using an illegal parameter. Examples: square root of a negative
    argument, negative matrix dimension, negative or zero LOG arguments,
    SET (X,Y) with one value out of bounds, etc. Or USR call without
    first POKEing the entry point.

FD Bad File Data: Data input from an external source (i.e., tape) was
    not correct or was in improper sequence, etc.

ID Illegal Direct: The use of INPUT as a direct command, without a
    statement number.

L3 DISK BASIC only: An attempt was made to use a statement or function
    which is available only when the TRS-80 Mini Disk is connected via
    the Expansion Interface.

LS String Too Long: A string variable was assigned a string value which
    exceeded 255 characters in length. Break it into two or more strings.

MO Missing Operand: An operation was attempted without providing one of
    the required operands.

NF NEXT without FOR: NEXT is used without a matching FOR statement.
    This error may also occur if NEXT *variable* statements are reversed
    in a nested loop.

NR No RESUME. End of program reached in error-trapping mode.

OD Out of Data. A READ or INPUT # statement was executed with insuffi-
cient data available. DATA statement may have been left out or all
data may have been read from tape or DATA.

OM Out of Memory. All available memory has been used or reserved. This
may occur with very large matrix dimensions, nested branches such as
GOTO, GOSUB, and FOR...NEXT loops, or with large strings. Usually the
program can be rewritten so that it uses less memory. See separate
suggestions.

OS Out of String Space. The amount of string space allocated was ex-
ceeded. Usually this can be fixed by starting program with CLEAR n
for some $n > 50$ or by using CLEAR MEM/2.

OV Overflow. A value input or derived is too large or small for the
computer to handle.

RG RETURN without GOSUB. A RETURN statement was encountered before a
matching GOSUB was executed.

RW RESUME without ERROR. A RESUME was encountered before ON ERROR GOTO
was executed.

SN Syntax Error. This usually is the result of incorrect punctuation,
open parenthesis, an illegal character or a misspelled command.

ST String Formula Too Complex. A string operation was too complex to
handle. Break up operation into shorter steps.

TM Type Mismatch. An attempt was made to assign a non-string variable
to a string or vice-versa.

UE Unprintable Error. An attempt was made to generate an error using an
ERROR statement with an invalid code.

UL Undefined Line. An attempt was made to refer or branch to a nonexis-
tent line.

/∅ Division by Zero. An attempt was made to use a value of zero in the
denominator.

USING KEYBOARD COMMANDS IN YOUR PROGRAMS

    TRS-80 Level II BASIC has an unusual extra "goodie" you may find handy. Commands such as RUN, NEW, LIST, DELETE 100-499, and similar keyboard commands can be inserted into and executed from a program. Try the following programs to get the feel of the idea.

```
100 CLS
110 FOR K=1 TO 30
120    PRINT K;
130 NEXT K
140 PRINT
RUN
```

Then add

```
150 RUN
```

and RUN it again.

Now, change 150 to

```
150 LIST
```

and RUN it again.

Depress BREAK and change 150 to

```
150 DELETE 100
```

LIST the program. RUN it, then LIST it again. Note that 100 has disappeared.   Change 150 to 150 NEW.

and RUN it again.

    It seems rather straightforward until you discovered that if you re-enter the program 100 to 140 and add

```
150 LIST
160 RUN
```

the program never gets to 160. Most commands (RUN is an exception.) send the computer back to READY mode instead of continuing the program. You can overcome this by using

    PRINT @ 970, "PLEASE TYPE   RUN (ENTER)."

just before the DELETE or LIST command.

    Try the program on the following page.

```
1Ø9 CLS
11Ø FOR K=1 TO 3Ø
12Ø    PRINT K;
13Ø NEXT K
14Ø PRINT : PRINT
15Ø PRINT "PLEASE TYPE   RUN (ENTER)"
16Ø DELETE 1Ø9--16Ø
165 PRINT : PRINT : PRINT
17Ø PRINT "WHEN THIS IS LISTED NOW, LINES 1Ø9 TO 16Ø WILL BE MISSING."
18Ø PRINT : PRINT
19Ø LIST
RUN ENTER
```
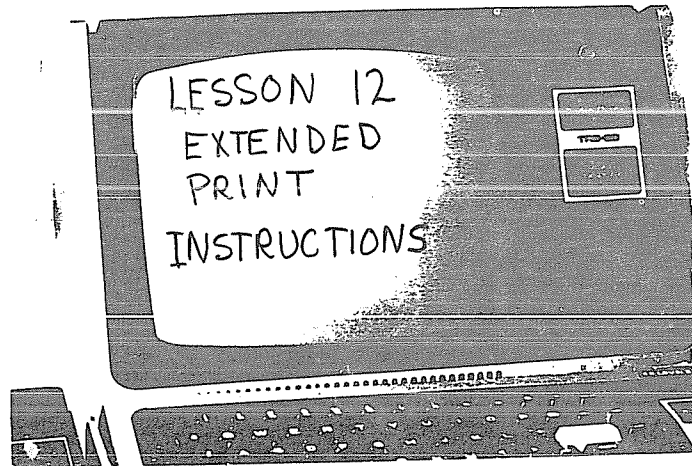
Advanced programmers use this technique to get extra space when
OS ERRORS plague them. Early parts of the program can be deleted before
the string variables are used. A sample might be:

```
1Ø
.
.
.
1ØØ
.
.
.
499
5Ø4 REM    TRANSITION TO GET MORE STRING SPACE
5Ø5 CLS
51Ø PRINT : PRINT
515 PRINT "PLEASE TYPE   RUN(ENTER)"
52Ø DELETE 1Ø-499
6ØØ REM    MAIN PROGRAM USING STRINGS STARTS HERE
61Ø CLEAR 5ØØØ : DIM B$(4Ø), M$(5Ø)
62Ø PRINT "THIS IS THE MAIN STRING PROGRAM."
7ØØ LIST
```

LESSON 12
EXTENDED
PRINT
INSTRUCTIONS

You are already familiar with the instruction

PRINT

which gives a blank line and

PRINT A,B,C,D

which gives wide (4 column) spacing where a comma is used and close-packed spacing where a semicolon is used.

PRINT A; B,C,D

is also familiar (see Lesson 2).

BASIC has several other useful instructions including

PRINT TAB( )
PRINT USING A$,K
PRINT STRING$(K,"#")

EXTENDED PRINT INSTRUCTIONS

PRINT TAB (expression)

The "expression" may be a constant or a variable or a computed expression which is an integer between Ø and 255. This "tabs" the cursor to the indicated position. It is <u>not</u> possible to move the cursor to the left by tabbing. If the cursor is already to the right of the TAB expression, the TAB is ignored. Any indicated printing occurs at the place where the cursor is.

Try the following:

1ØØ   A = 3

11Ø   B = 5

12Ø   PRINT TAB(A) A; TAB(B) B; TAB(A+B) "A+B"; TAB(35) "35";
      TAB(1Ø*B+A) "Z"

on your TRS--80.

The TAB(   ) function is particularly useful in graphing and the creation of special tables and forms.

PRINT USING *string;value*

It is possible to format the output in a more precise manner than is available on the ordinary PRINT instruction employing   ; , and " " or even the PRINT @ instruction.

The PRINT USING instruction makes this possible. Try the following program.

```
  10   CLS
 900   PRINT  "PLEASE TYPE YOUR NAME, INCLUDING MIDDLE INITIAL"
 910   INPUT N$
1000   FOR K = 917 TO 1130 STEP 13.25
1010     PRINT  "PAY TO THE ORDER OF   "; N$; "   ";
1020     PRINT USING "**$###.## DOLLARS"; K
1050   NEXT K
```

In instruction 1020, the "**$###.## DOLLARS" gives the format in which the material is to be printed. The **$ says to fill up any space before the first digit with asterisks and then a $. The decimal point followed by two #-signs (.##) indicates exactly two places beyond the decimal point are to be printed. Note the change in number of * where the amount changes from $996.50 to $1009.75 in the output when you run this program.   Consult your TRS-80 Level II BASIC Reference Manual, Chapter 3 for additional information.

PRINT USING statements may use any of the following field specifiers:

Numeric Character	Function	Example
#	Numeric field (one digit per #).	###
.	Decimal point position.	##.###
+	Print leading or trailing sign (plus for positive num- bers, minus for negative numbers).	+#.### #.###+ -#.### #.###-
--	Print trailing sign only if value print- ed is negative.	###.##-
**	Fill leading blanks with asterisk.	**###.##

(Continued next page)

Numeric Character	Function	Example (*continued*)
\$\$	Place dollar sign immediately to left of leading digit.	\$\$####.##
**\$	Dollar sign to left of leading digit and fill leading blanks with asterisks.	**\$####.##
↑↑↑↑	Exponential format, with one significant digit to left of decimal.	#.######↑↑↑↑

String Character	Function	Example
!	Single Character	!
%*spaces*%	String with length equal to 2 plus number of spaces between % symbols.	% %

PRINT STRING$ (K,"CHARACTER" or NUMBER)

This is very useful in graphing since it returns a string of K characters each of which is the character enclosed in quotation marks. It is also possible to use the ACSII number associated with a given character. The value K must be between 0 and 255, as must any number used for ASCII characters. See Lesson 4 for list of ASCII equivalents.

PRINT STRING$ (25, "#") produces

######################### (25 of them)

on your screen.

The following program plots the number K followed by a string of K^2 symbols:

```
100   CLS
110   FOR K = 1 TO 10
120      L = K*K
130      PRINT K; STRING$(L,"%")
135      PRINT
140   NEXT K
150   GOTO 150
```

Try it on your screen and see what happens. Then change the symbol inside the "   " to some other symbol of your choice.

Lesson 4 included a program to generate a bar graph (histogram).

```
  5   CLEAR 200
 10   CLS
100   PRINT @ 6, "GRAPH OF ENROLLMENTS AT THE UNIVERSITY OF OKLAHOMA"
110   READ Y,D
115   IF Y < 0 THEN 115
120   DATA 1940,21,1950,28,1960,32,1970,51,1980,59,-1,-1
130   PRINT Y; STRING$(D,143)
140   GOTO 110
```

The instruction
   5 CLEAR 200 clears all variables and sets aside 200 bytes for string
               variable use. This is maybe more than we need.

110 READ Y,D reads two values from the DATA string.

Instruction

130   PRINT Y; STRING$(D,143) prints the current value of Y (read in 110
      from DATA) and follows this with a D-long string of the symbol
      having ASCII equivalent of 143 which is

The same bar graph results as in the Lesson 4 program using SET (X,Y):
SET (X,Y+1), but in much less time and with a shorter program. The DATA
statement is also easy to change in the new program.

Note that if your D value is greater than 59, the bar will be continued
on a second line. However, D may be as great as 256 if desired, but
you'll need to use CLEAR 25Ø if D is that large.

Here is another program that may interest you. Try it out your-
self.

```
1ØØ   CLEAR 63
11Ø   CLS
12Ø   FOR Y = Ø TO 96Ø STEP 64
13Ø       PRINT @ Y, STRING$(63,191)
14Ø   NEXT Y
15Ø   GOTO 15Ø
```

After you see what it does, run the same program with 1ØØ changed to

1ØØ   CLEAR 6Ø   {Why did you get OS ERROR, if you did?}

You may also wish to change to 1ØØ CLEAR 1ØØ and then change STRING$
(63,191) to STRING$(64,191).   See if you can explain the unexpected
dark band at the bottom of the screen when you RUN it.

Sometimes it is handy to be able to clear only a portion of the
screen. Try this:

```
1Ø9 CLS
11Ø FOR K=1 TO 2Ø5
12Ø     PRINT K;
13Ø NEXT K
14Ø PRINT
2ØØ PRINT  @   256, CHR$(3Ø); "THIS IS THE NEW LINE FROM STMT 2ØØ.";
3ØØ PRINT  @   512, CHR$(3Ø); "THIS CAME FROM STATEMENT 3ØØ.";
5ØØ GOTO 5ØØ
RUN
```

Depress BREAK to regain control.

After you have RUN this program a couple of time with the semi-
colon at the ends of lines 2ØØ and 3ØØ, remove one of the semicolons and
RUN the revised program.
Investigate what happens if you insert:

  299 PRINT CHR$(23);

You may also wish to use CHR$(31) in statement 3ØØ to see what happens.
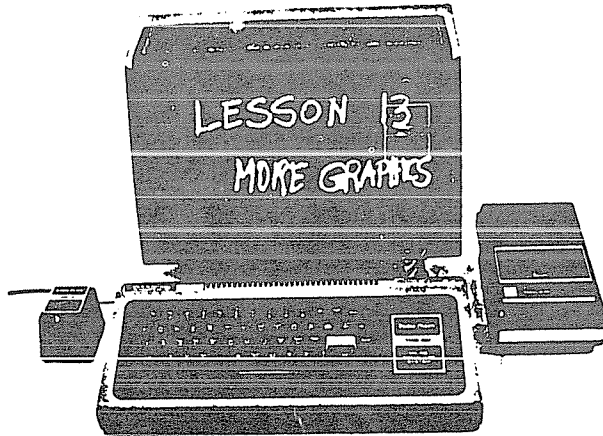
More Graphics

Lesson 4 provided an introduction to graphics that may be adequate for your needs. However, you should know that the TRS-80 has many graphic abilities in addition to the

CLS	clears screen
SET (X,Y)	turns on rectangular spot at (X,Y)
RESET (X,Y)	turns off rectangular spot at (X,Y)
POINT (X,Y)	tests to see if point (X,Y) is turned on, if so it returns a logical TRUE (-1), otherwise a logical FALSE (∅). Can be used in IF instructions (See Lesson 14, for details.) on logical operators.
CHR$(N)	produces actions, letter or symbol that corresponds to code N where $0 \leq N \leq 255$

For example: the operator ASC(*symbol*) is the inverse of CHR$(N). It produces the numeric (ASCII) code corresponding to the symbol in parentheses (the first symbol in the string, if the parentheses contain a string of more than one symbol). This is used in an excellent cipher program considered in Lesson 14 of this text.

In Lesson 4 you ran the following program:

```
10    CLS
100   FOR K = 33 TO 191
110      PRINT K,
120      FOR L = 1 to 45
130         PRINT CHR$(K);
140      NEXT L
150      PRINT
170   NEXT K
200   GOTO 100
```

If your TRS-80 shows both upper and lower case letters (a modification you may obtain, if needed) CHR$(K) for K=65 to 90 will display upper case letters A to Z while K=96 to 127 displays the corresponding lower case (SHIFT) symbols corresponding to @, A to Z, +, +, ←, →, —. Most TRS-80's display only upper case letters for both ranges.

Run it again now. Change instruction 100 to

     100 FOR K = 20 TO 255

and run it again, after turning to the table given in Lesson 4 that
discusses the effect of CHR$(N) for various N values.

The pixel-blocks that appear corresponding to

     PRINT CHR$(K)

for K between 129 and 191 are particularly useful in speeding up graphics.
At first glance they may seem haphazard, but actually the choice of num-
bers associated with the graphic you wish to light up is both logical and
easy to use, once it is explained. Each large block (letter & space be-
tween lines) is divided into six small rectangles (pixels)

The individual pixels are numbered.

1	2
4	8
16	32

To light up the pixels numbered 1,2,8 and 16,

add the sum of pixels used: 1 + 2 + 8 + 16 = 27

and add that sum to 128 giving

     128 + (1 + 2 + 8 + 16) = 155.

The instruction

     PRINT CHR$ (155)

will light up the   desired pixels much faster than

will SET (X,Y).

Much time can be saved in graphics by using CHR$( ).
You may use the instruction

POKE *address, pixel number*

where *address* is 15360 plus the number between 0 and 1023 which locates.
the position (see Lesson Four   ) of the 6-block under consideration.
The *pixel number* is the number (128 + (sum of desired pixels)).

For example:

The entire 6-block

1	2
4	8
16	32

has sum 63 so 128 + 63 = 191 is

the pixel number to turn on the entire 6-block.

The program

```
100  CLS
110  FOR X = 15360 TO 16383
120     POKE X, 191              (See Lesson 16 for POKE instruction.)
130  NEXT X
140  GOTO 140
```

RUN

will "white out" the entire screen in record time. It can be combined
with:

```
140  X = 2 + RND(121) : Y = 1 + RND(43)
150  RESET (X,Y) : RESET(125-X,Y)
160  RESET (X,46-Y) : RESET(125-X,46-Y)
170  GOTO 140
```

to produce a reverse art pattern, if desired.

     Try some additional experiments with the above 100-140 program such
as combining it with

```
140  PRINT @ RND(1000), "HI your name";
150  IF RND (0) >.01 THEN 140 ELSE 100
```

or some other experiments of your own design. Remember as long as you
keep beverages away from the TRS-80 and do not abuse it, you probably will
not harm it by your programming experiments -- and the best way to learn
computing is to compute and then to think about the results before
continuing.

If you need to insert a variable number of blanks in a line

PRINT CHR$ (192+k)

will insert k blanks for k = 0, 1, 2,...,63.

This may be used to good effect in graphing by using

PRINT CHR$ (192 + X); "+"

as in the following program to graph a portion of X = .062Y^2 which lies in the first quadrant.

```
104  PRINT "GRAPH OF X = .062*Y*Y"
110  Y = 30
120  X = INT(.062*Y*Y)
130  PRINT X; CHR$(192 + X); "+"
150  Y = Y -2
160  IF X >2  THEN 120 ELSE 160
```

Note that it will be necessary to depress $\boxed{\text{BREAK}}$ to get the computer's attention after the graph is plotted.(WHY)?

If we had not created the "tight loop" by using ELSE 160, then when X got to be < = 2 the display screen would have inserted

READY
>—

onto the screen and scrolled the top off the screen. If you are not sure of this,

change 160 to 160 IF X >2 THEN 120

and reRUN the revised program.

The second edition of the LEVEL II BASIC Reference Manual (appendix C/6) shows a map of the graphic pixel blocks and their corresponding numbers. Both the character spacing (16 lines of 64 characters)numbered from 0 to 1023 and the pixel locations for use with SET (X,Y) (48 lines with $0 \leq Y \leq 47$ of 128 pixels with $0 \leq X \leq 127$) are shown mapped onto the grid. In each case the numbering starts in the upper left corner with 0 or (0,0).

If you remember the diagram on the right and use 128 + (pixels to be lighted), you will not need the reference.

1	2
4	8
16	32
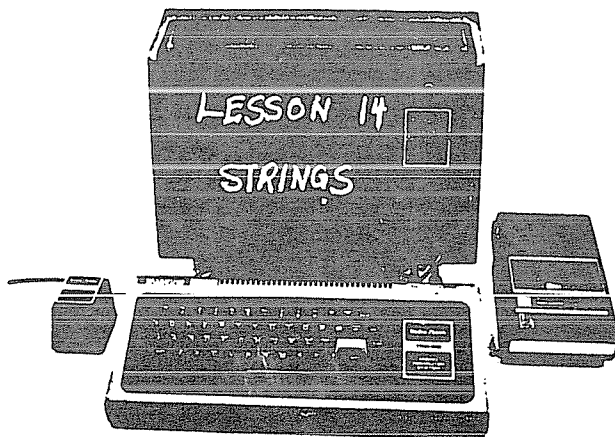
STRING FUNCTIONS

Let's re-examine the program of Lesson 4.

```
10   CLS
100  INPUT "PLEASE TYPE YOUR NAME"; N$
110  PRINT:PRINT:PRINT
120  PRINT "HELLO ";N$;". ": PRINT"      I HOPE YOU ARE ENJOYING THIS
                                                  COURSE."

125  FOR Q = 1 to 500:NEXT Q
127  PRINT CHR$(23)
130  GOTO 110
```

Line 10 clears the screen.
Line 100 prints what is in quotes and then awaits your input to string
variable N$.
Line 110 prints 3 blank lines (i.e., moves the next output line down
   3 rows).
Line 120 prints HELLO, followed by whatever you typed into N$, and then
on the next line prints

      I HOPE YOU ARE ENJOYING THIS COURSE. with a nine-space indentation,
if you allowed nine spaces.
Line 125 is a "timewaster". It forces the computer to count from 1 to
500 by ones before continuing.
Line 127 changes all output to double-size until the next CLS instruction
is executed. This eliminates half of the material currently on the
screen, but new material will be printed in double-wide characters.
Line 130 sends the program back to line 110 to print three more blank
lines followed by HELLO your name (double-size this time) etc., etc.,.....
until you depress the BREAK key to stop it.

That wasn't too hard was it?

Play around a bit with the messages in line 12Ø and change them to suit
your whim--be sure to include N$ as part of your output--note that N$ is
a variable and must be outside of the quotes, separated from the quotes
by semicolons.

   String variables use up quite a lot of memory space--not as much as
on many computers, since the TRS-80 LEVEL II string variable uses only
as much space as it needs--i.e. short strings use less memory than long
strings. On many computers, string variables require the same (maximum)
space even if not all is needed. If you have a 4-K Level II, you may run
out of memory if you use many variable strings. The following program
is designed to create very simplistic sentences and runs on 4-K. Note
that string variables may be dimensioned like floating-point variables.


```
2     CLEAR 1ØØØ
5     CLS
1Ø    DIM N$(7), V$(2)
1ØØ   PRINT "PLEASE TYPE 8 NOUNS."
11Ø   FOR K = Ø to 7
12Ø      PRINT K + 1, :INPUT N$(K)
13Ø   NEXT K
2ØØ   PRINT "TYPE 3 VERBS.  PLEASE USE DIFFERENT TENSES."
21Ø   INPUT "1 ";V$(Ø)
22Ø   INPUT "2 ";V$(1)
23Ø   INPUT "3 ";V$(2)
5ØØ   PRINT "THAT IS FINE.  NOW I'LL CREATE SOME SIMPLE SENTENCES."
6ØØ   PRINT N$(RND(8)-1);" " ; V$(RND(3)-1);" ";N$(RND(8)-1); " . "
62Ø   GOTO 6ØØ
```

Put it on your computer and RUN it. If you should run out of memory
(indicated by OM ERROR or OS ERROR) see if you can figure out what to do
about it (see Lesson 11).

   It is also feasible to store the words in DATA statements. In this
case they are read into the program using a READ statement as the follow-
ing program, designed with tongue-in-cheek to generate impressive
phrases for use in reports and grant requests.

READ W$(I) is like INPUT W$(I), except that it obtains the string value
for W$(I) from a DATA statement instead of from the keyboard.

   The individual string entries in the data statement are each enclosed
in quotes and entries are separated by commas. The computer will go to
the next DATA statement if the DATA statement on which it is working is
exhausted. DATA may also read from tape--but not here.

DATA statements may also be used to store numerical data. In this case numerical variables (no $) are used in the READ statement and quotes are not used in the DATA statement.

```
1Ø   DIM W$(35)
2Ø   FOR I = Ø TO 35
3Ø      READ W$(I)
4Ø   NEXT I
5Ø   DATA "FUNDAMENTAL", "BASIC", "INTUITIVE", "STUDENT-CENTERED",
     "HOMOGENEOUS", "MODULAR"
51   DATA "PARALLEL", "CENTRAL", "EQUAL", "SUPERIOR", "COLLECTIVE",
     "JUDGMENTAL"
52   DATA   "ACCOUNTABLE"   "UNGRADED", "NON-SEXIST", "BEHAVORIAL",
     "SCIENTIFIC", "HUMANITARIAN"
53   DATA "INTRINSIC", "FOUNDATION", "SUPPORTIVE", "LIMITED", "UNIFORM",
     "DISADVANTAGED"
54   DATA "STRUCTURE", "PERFORMANCE", "REINFORCEMENT", "GROUPING",
     "CURRICULUM", "ENVIRONMENT"
55   DATA "OBJECTIVE", "BOARD", "EXPERIMENT", "POLICY", "TEACHING",
     "EXAGGERATION"
1ØØ  PRINT W$(RND(12)-1);"   ";
11Ø  PRINT W$(RND(12)+11);"   ";
12Ø  PRINT W$(RND(12)+23)
13Ø  PRINT
14Ø  FOR Q = 1 TO 6ØØ : NEXT Q
15Ø  GOTO 1ØØ
```

It is easy to substitute words of the reader's choice by changing the DATA statements (5Ø,51,52,53,54,55). The program chooses its first word from the first 12 words of the data list, the second word from words 13-24 and the third word from words 25-36 in the DATA list. You may be interested in seeing what happens if you replace one of the words between quotes with a blank space between quotes.

A SECOND SENTENCE GENERATOR

Let's write a program to generate more sentences. Consider the sentence:

    HELEN QUICKLY SAW THE RED FOX.

The parts of speech are

    NOUN   ADVERB   VERB   ARTICLE   ADJECTIVE   NOUN

Another sentence having the same pattern is:

    JOHN SADLY SWEPT THE DIRTY FLOOR.

Let's write a program that will accept 10 nouns, 5 adverbs, 6 verbs and
7 adjectives. We shall start by setting aside the needed space by using
 CLEAR and   DIMension statements.

```
  2  CLEAR 1000
  5  CLS
 10  DIM N$(9), A$(4), V$(5), J$(6)
 20  S$ = " "
```

Next, we shall write a portion of the program to permit us to type in the
words of our choice.

```
100  PRINT "TYPE  TEN  NOUNS.  DEPRESS    (ENTER)   AFTER EACH WORD."
110  FOR K = 0 TO 9
120     PRINT K+1;
130       INPUT N$(K)
140  NEXT K
150  PRINT
```

That wasn't so hard was it? The computer numbers the stored words 0 to 9,
but humans seem to prefer 1 to 10, so that is what we used for each.
Let's continue.

```
200  PRINT  "PLEASE TYPE 5 ADVERBS.  DEPRESS    (ENTER)   AFTER EACH WORD."
205  PRINT "ADVERBS USUALLY END IN -LY."
210  FOR K = 0 TO 4
220     PRINT K + 1;
230     INPUT A$(K)
240  NEXT K
250  PRINT
300  PRINT  "PLEASE TYPE 6 VERBS (PAST TENSE).  DEPRESS  (ENTER) AFTER
     EACH WORD."
310  FOR K = 0 TO 5
320     PRINT K + 1;
330     INPUT V$(K)
340  NEXT K
350  PRINT
```

```
400   PRINT  "PLEASE ALSO TYPE 7 ADJECTIVES.  DEPRESS  (ENTER)  AFTER
      EACH WORD."
410   FOR K = 0 TO 6
420      PRINT K+1
430      INPUT J$(K)
440   NEXT K
450   PRINT
500   PRINT  "THAT IS FINE .  NOW, I'LL CREATE SOME SENTENCES FOR YOU."
600   FOR Q = 1TO 1000 : NEXT Q
610   PRINT
620   PRINT   N$(RND(10)-1); S$; A$(RND(5)-1); S$; V$(RND(6)-1); S$; "THE"
      ; S$; J$(RND(7)-1); S$; N$(RND(10)-1); " . "
630   GOTO 600

RUN   ENTER
```

A list of the major STRING instructions follows:

Function	Operation	Examples
ASC(*string*)	Returns ASCII code of first character in string argument.	ASC(B$) ASC("H")
CHR$(*code exp*)	Returns a one-character string defined by *code*. If *code* specifies a control function, that function is activated. (See Lesson 4 for equivalent list.)	CHR$(34) CHR$(K)
FRE(*string*)	Returns amount of memory available for string storage. Argument is a dummy variable.	FRE(A$)
INKEY$	Strobes keyboard and returns a one-character string corresponding to key pressed during strobe (null string if no key is pressed). Usually used as	INKEY$

```
300  A$ = INKEY$ : IF A$="" THEN 300 ELSE PRINT A$;
```

NOTE: There is no space between the quote marks. Thus IF A$ is the null string, the program loops back to strobe the keyboard again. ELSE it prints the value typed and continues with the next instruction.

Function	Operation	Examples
LEN(*string*)	Returns length of *string*(zero for null string).	LEN(A$+B$) LEN("HOURS")

Function	Operation	Examples
LEFT($string,n$)	Returns first n characters of $string$.	LEFT\$(A\$,1) LEFT\$(L1\$+C\$,8) LEFT\$(A\$,M+L)
MID\$($string,p,n$)	Returns substring of $string$ starting at position p in $string$ and containing the next n characters.	MID\$(M\$,5,2) MID\$(M\$+B\$,P,L-1)
RIGHT\$($string,n$)	Returns last n characters of $string$.	RIGHT\$(NA\$,7) RIGHT\$(AB\$,M2)
STR\$($numeric\ exp$)	Returns a string representation of the evaluated argument. This converts numeric variables to string variables.	STR\$(1.2345) STR\$(A+B*2)
STRING\$(n,"$char$")	Returns a sequence of n "$char$" symbols using first character of $char$. Useful in creating borders or divisions of the output.	STRING\$(30,".") STRING\$(25,"A") STRING\$(5,C\$)
VAL($string$)	Returns a numeric value corresponding to a numeric-valued string. This is the inverse of STR\$( ).	VAL("1"+A\$+"."+C\$) VAL(A\$+B\$) VAL(G1\$)

($string$ may be a string variable, expression, or constant.)

To <u>concatenate</u> strings (put them together), use the + operator as in

"2" + "YOU" + "76" = 2YOU76

Note that it is important to include desired leading and trailing spaces inside the quotes if you wish them to appear both in cancatenation and in PRINT "YOUR NAME IS   ";N\$; " ."

A CIPHER PROGRAM

    The following program creates a simple, but reasonably secure cipher by first changing each character into its numerical ASCII equivalent, using the ASC( ) instruction, adding a computer-generated random number to the ASCII code, moving the result to the desired range $48 \leq N \leq 90$, and then changing the modified ASCII code into an appropriate symbol using CHR\$( ). Decipherment is done using the reverse process. You may change the basic key by changing instruction 135 to R = Q*R for some other appropriate Q-value. But don't make the change unless you understand random-number generators. The choice of Q is critical and depends upon the number system used.

```
1Ø   DIM C(5ØØ)
4Ø   L = Ø
5Ø   INPUT "PLEASE TYPE YOUR KEY--POSITIVE TO ENCIPHER, NEGATIVE TO DE-
     CIPHER.";K
6Ø   R = ABS(K) : R1 = R
7Ø   PRINT "PLEASE TYPE COMPLETE MESSAGE FOLLOWED BY THE SYMBOL ↑."
1ØØ  P$ = INKEY$ : IF P$ = "" THEN 1ØØ ELSE PRINT P$;
1Ø5  IF P$ = "/" THEN 1ØØ
112  IF P$ = "↑" THEN 2ØØ
115  L = L+1
117  IF K< Ø THEN R1 = -1*R1
12Ø  C(L) = ASC(P$)+R1
125  IF C(L)> 9Ø THEN C(L) = C(L) - 43
13Ø  IF C(L)< 48 THEN C(L) = C(L) + 43
135  R = 197*R
137  R = R-1ØØØØ*INT(R/1ØØØØ)
139  R1 = INT(.ØØ4*R)
15Ø  GOTO 1ØØ
2ØØ  PRINT
2Ø5  IF K < Ø THEN 3ØØ
21Ø  FOR M = Ø TO L-1 STEP 5
215      FOR N = 1 TO 5
22Ø          PRINT CHR$(C(M+N));
23Ø          NEXT N
24Ø      PRINT "   ";
25Ø  NEXT M
26Ø  END
3ØØ  FOR M = 1 TO L
31Ø      PRINT CHR$(C(M));
32Ø  NEXT M
4ØØ  END
```

Note, that instruction 1ØØ uses the INKEY instruction which strobes the keyboard once. If a key is depressed during the strobe, that value is stored in variable P\$, otherwise P\$ contains a null (not a blank, just nothing at all). The instruction

$$\text{IF P\$ = "" \quad THEN 1ØØ ELSE PRINT P\$}$$

sends the computer back to 100 P$ = INKEY$ if P$ is blank, otherwise it prints the value of P$ (the key just depressed) on the screen and continues to the next instruction

105   IF P$ <="/" THEN 100

which essentially refuses any symbols before / (= 47 in ASCII--see Lesson 4).

112   IF P$ = "↑"   THEN 200

sends the computer to 200 when the message is complete, as indicated by typing   ↑

Lines 115 to 150 encipher or decipher the message depending upon whether the key K is positive (encipher) or negative(decipher). Each user will presumably have a different numerical key.

205   IF   K < 0   THEN 300

sends the computer to 300 if it is deciphering the message, and prints the entire message as one string (blanks between words were not enciphered--they could have been, but weren't).

IF K >= 0 then the computer continues with instruction 210 to 260 which displays the enciphered message in blocks of five symbols (a standard cipher practice).

ROBOT COUNSELOR

The following program, which runs on any 4K TRS-80, gives a brief insight into what programs that have more memory available can be expected to do.

```
1   REM       ROBOT COUNSELOR
2   RANDOM
5   CLS
7   CLEAR 100
10  DIM G$(9)
11  G$(0) = "HMM...MMM...,VERY INTERESTING"
12  G$(1) = "STRANGE.  NOT ABNORMAL, YOU UNDERSTAND, JUST STRANGE"
13  G$(2) = "FASCINATING"
14  G$(3) = "UNUSUAL, BUT PERFECTLY REASONABLE UNDER THE CIRCUMSTANCES"
15  G$(4) = "THERE MAY EASILY BE MORE TO IT THAN THAT"
16  G$(5) = "THAT MAY BE AN EXAGGERATION"
17  G$(6) = "VERY COMMON IN TODAY'S WORLD"
18  G$(7) = "BEWILDERING.  I DO NOT UNDERSTAND IT"
19  G$(8) = "I HOPE TOMORROW WILL BE BETTER"
20  G$(9) = "SURPRISING IN THE LIGHT OF YOUR BACKGROUND"
50  PRINT  "I'D LIKE TO HELP YOU.  MAYBE TOGETHER WE CAN WORK OUT YOUR
    PROBLEMS.  PLEASE ANSWER ME WHEN I ASK YOU QUESTIONS."
```

```
52   FOR Q = 1 TO 400 : NEXT Q
53   PRINT
55   PRINT "JUST TYPE YOUR ANSWERS, THEN DEPRESS THE WHITE (ENTER) KEY."
57   FOR Q = 1 TO 300 : NEXT Q
59   PRINT
60   INPUT "PLEASE TYPE YOUR FIRST NAME AND DEPRESS (ENTER) KEY.";N$
61   PRINT : PRINT : PRINT  "HI ";  N$
70   PRINT : PRINT
80   FOR Q = 1 TO 900 : NEXT Q
100  PRINT : PRINT  "HOW DO YOU FEEL, ",N$;"?  PLEASE ANSWER IN ONE OR TWO
     WORDS."
110  INPUT R$
115  FOR Q = 1 TO 500
120  PRINT:PRINT:PRINT "OH, I SEE.  THAT IS ";G$(RND(10)-1);"."
130  FOR Q = 1 TO 800 + RND(1000) : NEXT Q
139  PRINT
140  PRINT "WHO DO YOU THINK MAKES YOU FEEL ";R$;"?"
150  INPUT R2$
155  IF R2$ = "ME" THEN R2$ = "YOU ALONE"
156  IF R2$ = "MYSELF" THEN R2$ = "YOU YOURSELF"
157  IF R2$ = "YOU" THEN R2$ = "THE TRS-80"
160  PRINT:PRINT
163  FOR Q = 1 TO 100 + RND(500):NEXT Q
165  PRINT "WELL NOW, ";N$; ",THAT IS CERTAINLY "; G$(RND(10)-1);"."
170  FOR Q = 1 TO 500 + RND(600):NEXT Q
180  PRINT:PRINT "DO YOU REALLY BELIEVE ";R2$;"MAKES YOU FEEL ";R$;"?"
190  INPUT A$
200  IF A$ = "NO" THEN PRINT "WELL ";N$;", PLEASE BE FRANK WITH ME.  IT
     SEEMS ";G$(RND(10)-1);"." : GOTO 140
210  PRINT:PRINT:
220  FOR Q = 1 TO 10 + RND(1000):NEXT Q
225  PRINT "THAT IS ";G$(RND(10)-1);".  IT IS ";G$(RND(10)-1);"."
228  PRINT "THINK ABOUT IT A BIT."
229  PRINT:PRINT "IT IS INDEED ";G$(RND(10)-1);"." :PRINT:PRINT
230  PRINT "REALLY NOW, TELL THE TRUTH.  PLEASE BE FRANK WITH ME."
235  PRINT "IT SEEMS ";G$(RND(10)-1);"."
240  GOTO 100
```

After you have run the program a few times, change some of the phrases
stored in G$( ) in lines 11 to 20. See if you can produce a more inter-
esting ROBOT COUNSELOR program.

LOGICAL OPERATORS

Have you wondered how the computer handles IF instructions?

The instruction

IF X >13   THEN 42Ø ELSE 3Ø

examines the number stored in location X and if X is greater than 13, sends the computer to statement 42Ø for its next instruction but if X is less than or equal to 13 sends the computer to statement 3Ø for its next instruction.

The instruction

IF X >125   THEN X = 125

examines the variable X and if X is greater than 125, sets X = 125.

Actually the expression X >125 is evaluated as a <u>logical</u> <u>expression</u>.

If the current value of X makes X > 125 a TRUE statement, then the value of the <u>logical</u> <u>expression</u>

(X > 125) is −1.

If the current value of X makes X > 125 a FALSE statement, then the value of the logical expression (X > 125) is Ø.

You can examine this by running the program

```
1ØØ  Y = 25
11Ø  FOR K = 1 TO 8
12Ø  X = K * Y
13Ø  PRINT K,X,(X > 125)
14Ø  NEXT K
```

which produces the output

1	25	Ø
2	5Ø	Ø
3	75	Ø
4	1ØØ	Ø
5	125	Ø
6	15Ø	−1
7	175	−1
8	2ØØ	−1

The actual values (TRUE = −1, FALSE = Ø) used are unimportant at this stage, and may vary on different computers. (LEVEL I TRS-80's use TRUE = 1, FALSE = Ø   .)

If the value produced represents TRUE (i.e., -1) then the remaining statements on the IF line are carried out (possibly several statements separated by colons).

If the value produced represents FALSE (i.e., $\emptyset$) then the computer skips the rest of the instructions in that statement line (possibly several instructions) and goes to the next line or the ELSE statement for its next instruction.

It is possible to <u>combine</u> statements using logical operators:

AND   If both expressions are true   TRUE (=-1) otherwise FALSE (=$\emptyset$)

OR   IF either expression is true   TRUE (=-1) otherwise FALSE (=$\emptyset$)

NOT   Interchanges TRUE (-1) and FALSE ($\emptyset$).

EXAMPLE:

```
100   INPUT A,B
110   PRINT "A=";A,  "B=";B
120   IF A> B  THEN PRINT "A IS GREATER THAN B"
130   IF (A > 0) AND (A < 10) THEN PRINT " 0 < A < 10." ELSE PRINT "A<= 0 or
      A>= 10."
140   IF (B > 20) OR (B < -20) THEN PRINT "ABS (B)> 20."
150   IF ((A> 0) AND (B > 0))  OR ((A < 0) AND (B< 0)) THEN PRINT "A*B> 0."
      ELSE PRINT "A*B <= 0."
160   GOTO 100
```

Actually the AND, OR, NOT logical operations also may be used to test individual <u>bits</u> in the binary storage of TRS-80 words since the operations perform Boolean operations on the bits of numbers as well as on logical statements. Readers interested in such masking operations should first consult chapter 8 of the TRS-80 Level II BASIC REFERENCE MANUAL and then a text on Boolean (logical) algebra.

LESSON 15
WHERE TO
LOOK NEXT
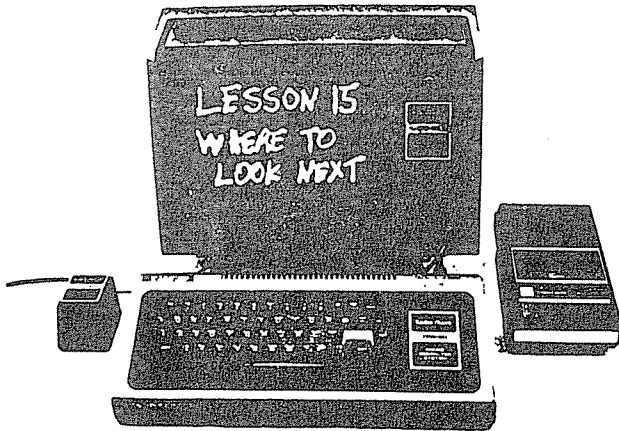
WHERE TO LOOK FOR ADDITIONAL INFORMATION

Your first source of additional information is the LEVEL I or
LEVEL II BASIC REFERENCE MANUAL that came with your TRS-80. You should
also be receiving (free) the Radio Shack Micro Computer Newletter if you
filled out the card requesting it that came with your TRS-80.

Fellow computer buffs are another excellent source of information.
If you are in real difficulty, don't hesitate to telephone or write to
The Radio Shack Computer Services Center in Ft. Worth. They maintain a
staff of knowledgable people who seem both willing and able to help TRS-
80 owners. The address is

        Computer Services
        900 Two Tandy Center   Telephone:
        Ft. Worth, TX 76102   1-(800)-433-1679

They do not assist in debugging or writing programs, but will glad-
ly assist you if you have questions about what a given BASIC or Z-80
instruction does or about the TRS-80 hardware.

Most computer programmers, either hobbyists or professionals, learn
most about computing by burning midnight oil at the computer. One learns
to compute by computing, and then by analyzing the results.

There seem to be clubs, conventions and magazines devoted to almost
any hobby that interests you, and computers are no exception. There are
even special clubs, conventions and magazines devoted exclusively to
microcomputers. Some are so highly specialized that they concentrate on
the TRS-80 microcomputer. Ask around your own area and see what micro-

computer clubs and/or conventions are available. If no microcomputer club exists in your area, organize one yourself.

No list of microcomputer magazines (journals) can ever be complete—new ones seem to start up almost monthly; others drop into oblivion. Some are excellent; others are a waste of funds. Most will send you a sample copy if you ask for one, then you can judge how well it fits your particular interests.

Here are some that we take, with our favorites indicated by a star ✿. Prices were current at time of printing.

Micro Computer Conference Proceedings

The Best of the West Coast Computer Faires, 333 Swett Road, Woodside, CA, 94062. Set of 4 $53; Vols I (1976) & III (1978) $13.72 ea.; Vols II (1977) & IV (1979) $14.78 ea.

✿ Show & Tell microCOMPUTER Conference Proceedings, Department of Mathematics, University of Oklahoma, Norman, OK, 73019. (1978) $7.50; (1979) $10; (1980) $10; (1981) $10. Conference held in May each year.

Micro Computer Periodicals

✿ Byte, Byte Subscriptions, P.O. Box 590, Martinsville, NJ, 08836, monthly, $15 per year. Both hardware and programs. Some technical material.

Computronics, TRS-80 Monthly News Magazine, P.O. Box 149, New York City, NY, 10956, monthly, $24 per year. Not worth the cost in my judgment—request an examination copy and judge for yourself. Magazines change.

The Computing Teacher, Computing Center, Eastern Oregon State College, La Grande, OR, 97850, bi-monthly, $8 per year.

✿✿ Creative Computing, P.O. Box 789-M, Morristown, NJ, 07960, monthly, $15 per year. Probably the best buy available. Something for everyone. Well written and authoritative articles.

Data General News, Data General Corporation, Southboro, MA, 01772 (free—devoted to their computers).

Dr. Dobbs Journal of Computer Calisthenics & Orthodontia, The Peoples Computer Co., Box E, Menlo Park, CA, 94025, monthly, $12 per year. Advanced and excellent.

✿ Games, Games Productions, Inc., 515 Madison Ave., New York City, NY, 10022, bi-monthly, $5.97 per year. Not computer oriented, but lots of good ideas you can use.

Games & Puzzles, Edu-Games (U.K.) Ltd., P.O. Box 4, London, England, N64DF

✿ Journal of Recreational Mathematics, Baywood Publishing, 120 Marine, Farmingdale, NY, 11735. Interesting problems, some computer related. Quarterly, $25. ($10 to individuals, home address, personal check.)

Personal Computing, Circulation, Personal Computing, 1050 Commonwealth Ave., Boston, MA, 02215, monthly, $14 per year.

✿ Computer Music Journal, ISSN 0148-9276 MIT Press Journals, 28 Carleton St., Combridge, MA 02142. Price $4 per issue, $20 yearly.

✿ PROG 80, Softside Publications, 17 Briar Cliff Dr., Milford, NH, 02055, monthly, $15 per year. Lots of goodies for the serious TRS-80 programmer.

Radio Shack Microcomputer Newsletter, Radio Shack, One Tandy Center, Fort Worth, TX, 76102, TRS-80 owners receive free for 6 months, then $12/year.
Recreational Computing, People's Computer Company, 1263 El Camino Real, Box E, Menlo Park, CA, 04025, 6 issues per year, $10.00.

The Recreational Programmer, Computer Software, P.O. Box 2571, Kalamazoo, MI, 49003, bi-monthly, $12 per year.

Softside, Softside Publications, 17 Briar Cliff Dr., Milford, NH, 02055, monthly, $15 per year. Lots of BASIC games in this one.

T-PAL, The Mail Mart, Box 11102, San Francisco, CA, 94101, monthly, $24 per year. Seems expensive for what you get. Ask for a sample copy, it should have improved.

✿ TRS-80 Computing, Computer Information Exchange, P.O. Box 158, San Luis Rey, CA, 92068, monthly, $15 per year. Lots of goodies here.

✿ 80-US, 80-NW Publishing, 3220 N. 32nd St., Tacoma, WA, 98407, sample copy, $3, $16 per year. Six issues annually.

PAGE, Bulletin of the Computer Arts Society, John Lansdown, 50151 Russell Square, London, WCIB4JX, England, quarterly, $51 per year (students, half-price). Or write Kurt Lauckner, Math. Dept., Eastern Michigan University, Ypsilanti, MI, 48917. Devoted to computer assisted art.

TRS-80 Users Group, 7554 Southgate Rd., Fayetteville, NC, 28304, monthly, $12 per year. An informal publication for amateurs that is
. well worth the price.

✿ 80 Microcomputing, Started in January 1980 and so far looks to be well worth the $12/year. This monthly slick magazine devoted to TRS-80 does not pull punches and keeps you informed on the latest updates and changes and evaluations of hardware and software related to the TRS-80.

Where to Buy Programs

We don't purchase many programs; we write our own because it is more fun. Most of the microcomputer magazines and conference proceedings carry programs. Some, you can obtain on tape at a modest cost. Most carry ads for programs that will run on your TRS-80. Some sell programs that are definite "rip-offs". Others are bargains. Few have adequate documentation with them. Still, you will almost certainly be tempted to buy some, just to try them out. Read the ads. Go to your local Radio Shack (or Radio Shack Computer Center, if you have one...many middle-to-large cities do). Go to any other computer stores in your area and ask to see the programs in action. (Don't let your local computer store disparage the TRS-80. Some will because they do not sell them.) Attend a microcomputer club in your area. Ask about available software.

If you haven't seen Leo Christopherson's Android Nim (with sound), you should. Nim isn't much of a game, but the animation is superb (1980 price, $14.95, runs on 16K Level II TRS-80). If you wish a TRS-80 chess or checker playing program, look over the results from the most recent national microcomputer chess or checkers tournament. There are a dozen programs available for under $20 each, and the quality varies considerably. Read the comments and the ads in whatever microcomputer periodicals you and/or your libaray take.

You should probably write to:

> People's Software
> Computer Information Exchange
> Box 158
> San Luis Rey, CA 92066

They put a couple of dozen programs on a tape and sell it for $8.00 (see Lesson 7). They also sell more expensive programs, like People's Pascal ($ 23.50). New programs are added frequently. Drop them a card asking for a current price list.

HAVE FUN! GROW WITH YOUR NEW HOBBY. READ, EXPERIMENT, DISCUSS YOUR EXPERIENCES. HAVE FUN!
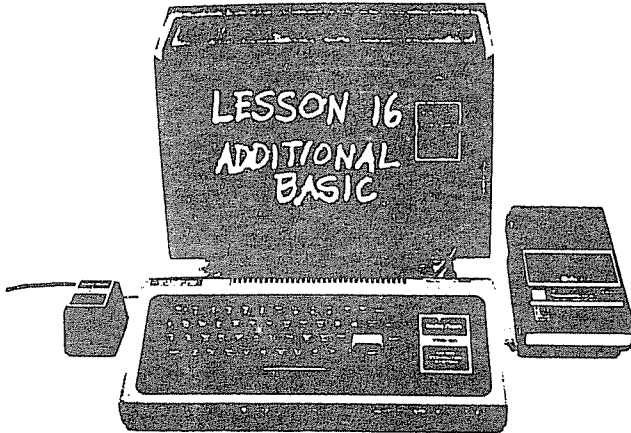
Your <u>TRS-80 Level II BASIC Reference Manual</u> contains details of the available Level II BASIC instructions. If you have an expansion inter-face, line printer, or disk you will find additional instructions defined in the accompanying reference manuals. Here are several additional Level II BASIC instructions you may find convenient.

Additional Keyboard Commands   Examples

AUTO *mm,nn*   AUTO   1ØØ   [ENTER]

Turns on automatic line numbering   AUTO   1ØØ,5   [ENTER]
beginning with *mm* and in steps of
nn. If ,*nn* is omitted, the default   AUTO   3ØØ, 20   [ENTER]
step size is 1Ø.

DELETE *mm-nn*   DELETE   41Ø   [ENTER]

Deletes program steps from line   DELETE   8Ø-23Ø   [ENTER]
mm to line *nn* inclusive. Both
mm and *nn* should be actual statement
numbers in the program.

LIST *mm-nn*   LIST   27Ø-4ØØ   [ENTER]

Lists program from line *mm* to   LIST   14Ø-   [ENTER]
line *nn*.   LIST -27Ø   [ENTER]

                                                        LIST   [ENTER]
RUN *mm*   RUN   [ENTER]

Executes program beginning at line *mm*.   RUN   45Ø   [ENTER]

SYSTEM   SYSTEM   [ENTER]

Enter monitor mode for loading Z-80
language program from cassette tape.
Frequently used with advanced com-
mercial programming.

Additional Program Instructions	Examples

TRON

TRON

Turns on TRACE program in which
variable values are shown on screen
to help you debug program. TRACE
program is always in the TRS-80, as
part of your Level II BASIC ROM.

TROFF

TROFF

Turns TRACE program off. The TRACE
program is still in the computer.

USR(n)

USR($\emptyset$)

Branches to a Z-80 machine language
subroutine already entered by the
user. You'll need to use POKE first.

RESTORE

RESTORE

Resets the DATA pointer to the first
DATA statement to permit program to
reuse the same data.

GOSUB nn

GOSUB 46$\emptyset$

Branch to the subroutine beginning
at line number nn.

GOSUB 37$\emptyset\emptyset$

RETURN

RETURN

Each subroutine must contain at
least one RETURN statement. This
sends the program back to the line
following the GOSUB that branched
to the subroutine.

ON α GOTO mm, nn, pp, qq, ' ' '

ON Z GOTO 2$\emptyset\emptyset$, 3$\emptyset\emptyset$, 4$\emptyset\emptyset$

ON K+2 GOTO 7$\emptyset\emptyset$,7$\emptyset\emptyset$,4$\emptyset\emptyset$,3$\emptyset\emptyset$,5$\emptyset\emptyset$

If the value of INT(α), where α is a
variable or expression, is one of the
numbers 1,2,3,4,' ' ',k then go to the
statement number in that position on
the list of statement numbers mm, nn,
pp, qq, ' ' '.

ON α GOSUB mm, nn, pp, qq, ' ' '

ON K GOSUB 33$\emptyset\emptyset$, 2$\emptyset\emptyset\emptyset$, 4$\emptyset\emptyset$

Same as ON α GOTO except the RETURN
pointer is set so program will return
to statement following ON α GOSUB....
when RETURN is encountered.

Arithmetic Functions

Function	Operation(unless noted otherwise, $-1.7E+38<=exp<=1.7E+38$)	Examples
ABS(exp)	Returns absolute value.	ABS(L*.7) ABS(SIN(X))
ATN(exp)	Returns arctangent in radians.	ATN(2.7) ATN(A*3)
CDBL(exp)	Returns double-precision represen-tative of exp.	CDBL(A) CDBL(A+1/3#) '
CINT(exp)	Returns largest integer not greater than exp. Limits: $-32768<=exp<+32768$.	CINT(A#+B)
COS(exp)	Returns the cosine of exp; assumes exp is in radians.	COS(2*A) COS(A/57.29578)
CSNG(exp)	Returns single-precision represen-tation, with 5/4 rounding in least significant decimal when exp is double-precision.	CSNG(A#) CSNG(.33*B#)
EXP(exp)	Returns the natural exponential, e^{exp}=EXP(exp).	EXP(34.5) EXP(A*B*C-1)
FIX(exp)	Returns the integer equivalent to truncated exp (fractional part of exp is chopped off).	FIX(A-B)
INT(exp)	Returns largest integer not greater than exp.	INT(A+B*C)
LOG(exp)	Returns natural logarithm (base e) of exp. Limits: exp must be positive.	LOG(12.33) LOG(A+B+B)
SGN(exp)	Returns -1 for negative exp; 0 for zero exp; +1 for positive exp.	SGN(A*B+3) SGN(COS(X))
SIN(exp)	Returns the sine of exp; assumes exp is in radians.	SIN(A/B) SIN(90/57.29578)
SQR(exp)	Returns square root of exp. Limits: exp must be non-negative.	SQR(A*A - B*B)
TAN(exp)	Returns the tangent of exp; assumes exp is in radians.	TAN(X) TAN(X*.01745329)

exp may be any numeric variable, constant or computed expression.

There are a number of other Level II BASIC instructions available for special use on your TRS-80. If your programming is advanced enough to need these extra instructions see the TRS-80 Level II BASIC Reference Manual and the reference manuals accompanying any additional equipment you may have (Disk, Lineprinter) etc.

Three examples are:

POKE *location, value*
Loads value into memory *location*. The arguments must be decimal numeric variables, constants or functions with $0 \leq value \leq 225$. Used in advanced programming to change memory content or display screen and with USR( )function. See Chapter 8 of Level II BASIC Reference Manual.

POKE 16396,23
will disable the BREAK key. This is sometimes used to prevent unsophisticated users from BREAK -ing into a program.

PEEK (*address*)
Returns the (decimal) value stored in the memory *address* specified. Also, makes values of Z-80 machine language variables available to BASIC programs. See Chapter 8 of Level II BASIC Reference Manual.

```
10  FOR K = 0 TO 12288
20  PRINT K;PEEK(K),
30  NEXT K
```

will display the location and contents of your Level II BASIC ROM. It won't mean much to you, but it will do it.

RANDOM
is frequently executed once near the beginning of a program to reseed the RND( ) instruction. Do not use RANDOM inside of a loop or where it will be used repeatedly in your program.

```
1  REM        program name
2  RANDOM
```
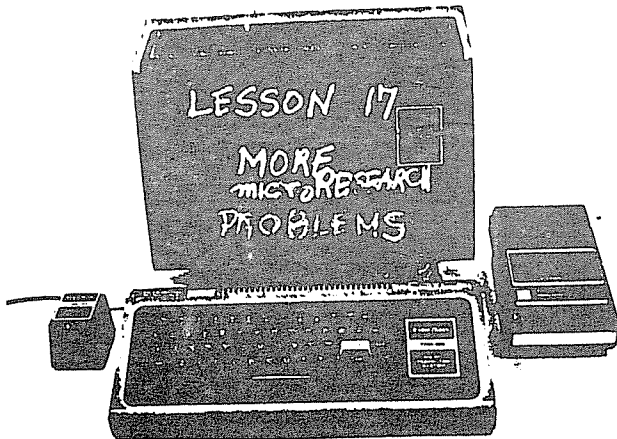
EXAMPLE 17-1

Let us examine a program designed to determine the factors of a given integer. Forms of this program have appeared in several magazines, books and even in tape form. The given program works. It produces the desired factors for any positive integer Z. However, it is easy to improve upon the program, as we shall see.

First examine the original program to see why it works.

```
 1Ø   REM   FIRST WORKING ATTEMPT TO FACTOR A NUMBER
1ØØ   INPUT "TYPE NUMBER TO BE FACTORED"; Z
11Ø   PRINT  "THE PRIME FACTORS OF";Z;"ARE";
12Ø   N = Z
13Ø   F = 2
14Ø   IF N/F < > INT (N/F) THEN 2ØØ
15Ø   PRINT F;
16Ø   N = N/F
17Ø   IF N = 1 THEN STOP ELSE 14Ø
2ØØ   F = F + 1
21Ø   IF F < = Z THEN 14Ø
```

The heart of the program is instruction 14Ø

IF N/F < > INT(N/F) THEN 2ØØ

which uses the integer function INT( ) to determine whether or not N/F is an integer (i.e., whether or not F is a factor of N). If F is not a factor of N, the program jumps to instruction 2ØØ which increases F by 1. If new F < =Z the program loops back to instruction 14Ø to determine whether or not the new F is a factor of N. If F is a factor of N, the value of F is printed, N is replaced by N/F and if the new N = N/F <> 1,

the test is repeated. When either of the conditions N = 1 or F > Z occurs, the original number Z is completely factored.

The program also uses an axiom many experienced programmers have found worthwhile

```
╔══════════════════════════════════════════════════╗
║           Don't alter the value of                ║
║                                                    ║
║    an INPUT variable inside your program           ║
╚══════════════════════════════════════════════════╝
```

Note that the program inputs the value of Z but then sets N = Z and works with N. We could have ignored this motto here, but many experienced programmers will tell you they stick to it in their own work. It avoids trouble.

The program produces the prime factors of any positive integer of six or fewer digits you put in. Try it and see. Then examine the program to see exactly how it works.

If you have any trouble following the program, make a chart of variables in the program and step the program through. If your INPUT value is Z = 12, your chart will contain

Z	N	F	PRINTED OUT
12			The prime factors of 12 are
	12	2	2
	6	2	2
	3	3	3
	1	STOP	

After you understand how the program works, you may wish to create a flowchart of the logic involved. Flowcharts are frequently used to express the logic involved in more complicated problems. This particular program is probably not involved enough to require a flowchart, but it is easy to "see" the flowchart of a simple problem.
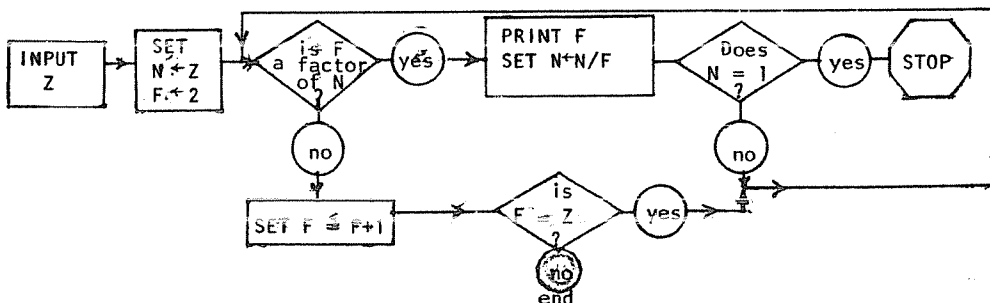
Your flow chart should look <u>something</u> like this:

```
                        ┌─────────────────┐
                        │   INPUT   Z     │
                        └─────────────────┘
                                │
                        ┌─────────────────┐
                        │ PRINT    MESSAGE│
                        └─────────────────┘
                                │
                        ┌─────────────────┐
                        │  SET N = Z      │
                        │  SET F = 2      │
                        └─────────────────┘
                                │
                                ▼
                            DOES
         (NO) ◄──────  N/F = INT(N/F)?  ──────► (YES)
          │                                        │
          ▼                                        ▼
    ┌──────────┐                            ┌──────────┐
    │   SET    │                            │ PRINT F  │
    │ F = F + 1│                            └──────────┘
    └──────────┘                                 │
          │                                       ▼
          ▼                               ┌──────────────┐
          IS                              │ SET N = N/F  │
 (NO) ◄ F ≤ Z ► (YES)                     └──────────────┘
  │               │                             │
  ▼               ▼                             ▼
 STOP                                        DOES
                                   (NO) ◄  N = 1  ► (YES)
                                     │             │
                                     ▼             ▼
                                                  STOP
```
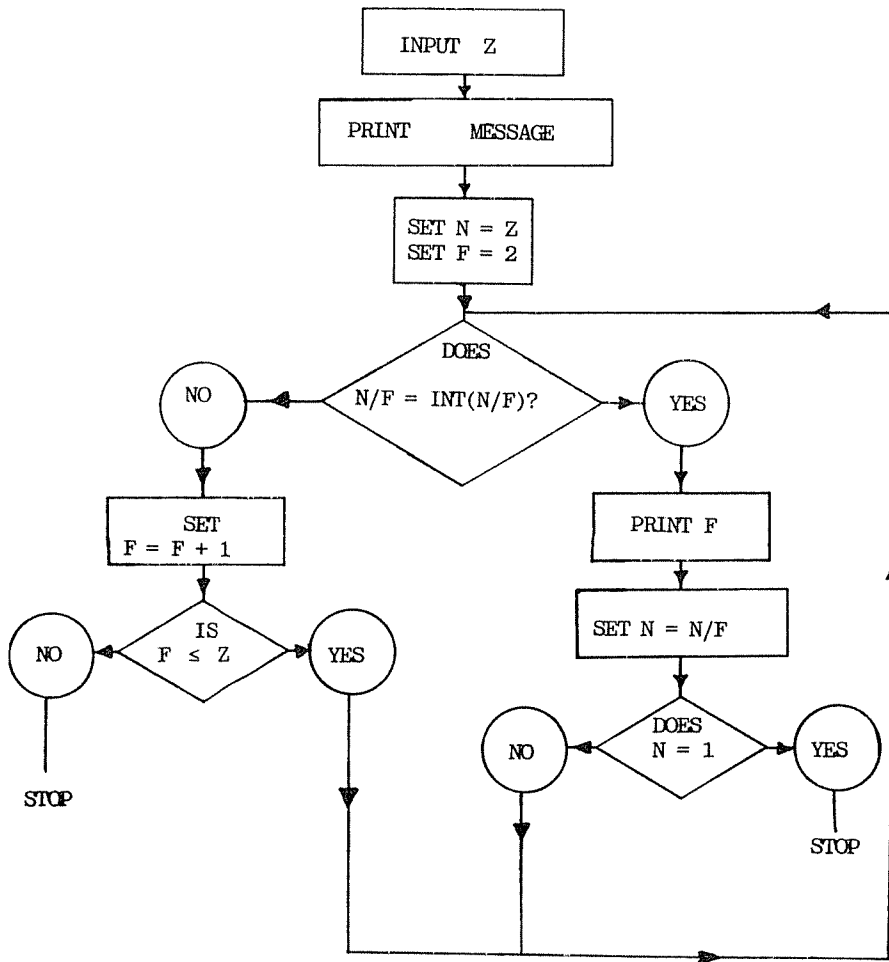
Note how easy it is to explain what the program is doing using a flow chart.

Actually, although the program in question has been published in several versions, it is an example of <u>deplorably poor programming practice</u>. It works fairly well on small values, but note the difference between Z = 1847 and Z = 1848 in the time required. Try it yourself before continuing.

Furthermore, Z = 1950 takes :01 second to factor and Z = 1951 takes 1:01 seconds to factor. This seems a long time for a powerful computer. Of course, it does do a lot of arithmetic in that time. In fact, that is just the trouble -- it does too much arithmetic. The program tests F = 2, 3, 4, 5, 6, 7, 8, 9,..., Z repeatedly. However, if we factor out all the 2's at the beginning, then there is no reason to also try 4, 6, 8, 10, etc. This should enable us to almost cut the time by half.

This can be done by making the following changes:

OLD PROGRAM

```
10  REM    FIRST WORKING ATTEMPT TO
    FACTOR A NUMBER
100 INPUT   "TYPE NUMBER TO BE
    FACTORED " ; Z
110 PRINT   "THE PRIME FACTORS
    OF "; Z; "ARE ";
120 N = Z

130 F = 2




140 IF N/F <> INT(N/F) THEN 200
150 PRINT F;
160 N = N/F
170 IF N = 1 THEN STOP ELSE 140
200 F = F + 1
210 IF F < = Z THEN 140
```

FIRST REVISION

```
10  REM    FIRST WORKING ATTEMPT TO
    FACTOR A NUMBER
100 INPUT   "TYPE NUMBER TO BE
    FACTORED " ; Z
110 PRINT   "THE PRIME FACTORS
    OF "; Z; "ARE ";
120 N = Z
125 IF N/2 <> INT(N/2) THEN 135
130 PRINT " 2";
132 N = N/2
134 GOTO 125
135 F = 3
140 IF N/F <> INT(N/F) THEN 200
150 PRINT F;
160 N = N/F
170 IF N = 1 THEN STOP ELSE 140
200 F = F + 2
210 IF F < = N THEN 140
```

In instruction

    210 IF F < = Z THEN 140

there is really no reason to try F values that are larger than the <u>current</u> value of N (instead of the original Z), so we <u>replace instruction 210 with</u>

    210 IF F < = N THEN 140

This should cut our time even further. Make the above changes in 125,

13Ø, 132, 134, 135, 2ØØ, and 21Ø. Then, try the program again.

N	Old Program Time	New Program Time
12	:01	:005
1847	:58	:30
1848	:01	:01
12345	:26	:14
-6	:01 *ignores sign*	:005
8.2	Runs through 21Ø and stops.	

Even the above program would not satisfy most professional programmers. The biggest complaints would probably be that the program malfunctions if negative values are used, and that the data was not tested to be sure the input value was an integer.

The problem of fractional values is readily overcome by inserting:

1Ø5 IF Z <> INT(Z) THEN PRINT "PLEASE USE INTEGER VALUES" : GOTO 1ØØ

Negative values could also be handled as improper input, but since they can be factored, we prefer another solution.

115 PRINT   "(" ; SGN(Z); ")" ,
12Ø N = ABS(Z)

This will produce either -1 or 1 in parentheses as a first factor. We include the parentheses since mathematically, neither -1, nor 1, is a prime. By ending instruction 115 with a comma, instead of a semicolon, we also provided extra space between the unit and the prime factors.

As long as we are improving our program, we might as well make full use of the TRS-80's arithmetic ability by requesting it to use DOUBLE PRECISION arithmetic, which permits us to work with numbers as large as 16 digits, instead of 6. We can do this by adding:

10 DEFDBL Z, N, F

However, if we do this, we will be using the much slower double precision arithmetic, even when our number to be factored has 6 or fewer digits. To avoid this, insert:
    1Ø DEFDBL Z
    1Ø6 IF Z > 999999 THEN DEFDBL N, F

Many programmers would prefer to print a blank line or two and loop the program back to the beginning rather than stopping it once the number is factored. This is easily done by changing instruction 17Ø to

    17Ø IF N=1 THEN 9Ø ELSE 14Ø

and adding:

```
 90 PRINT : PRINT
300 GOTO 90
```

Our revised program now reads:

```
 10 DEFDBL Z
 90 PRINT : PRINT
100 INPUT   "TYPE NUMBER TO BE FACTORED "; Z
105 IF Z <> INT(Z) THEN PRINT  "PLEASE USE INTEGER VALUES" : GOTO 100
106 IF Z > 999999 THEN DEFDBL N, F
110 PRINT   " THE PRIME FACTORS OF "; Z ; "ARE ";
115 PRINT   "(" ; SGN(Z); ")",
120 N = ABS(Z)
125 IF N/2 <> INT(N/2) THEN 135
130 PRINT "2";
132 N = N/2
134 GOTO 125
135 F = 3
140 IF N/F <> INT(N/F) THEN 200
150 PRINT F;
160 N = N/F
170 IF N = 1 THEN 90 ELSE 140
200 F = F + 2
210 IF F < = N THEN 140
300 GOTO 90
```

Although our new program takes more lines to write than the original
program, actually it produces results in less than half the time, and
works for many additional input values.

There are still three important cases to be considered. What happens
if Z = 0? Try it and see. If the output does not satisfy you, change the
program further by adding:

IF Z = 0 THEN PRINT " *some appropriate message.*"

Mathematically oriented readers may prefer to make the program even
faster by using only <u>prime</u> values for F instead of 2, 3, 5, 7, 9, 11,
13, 15, ...Z. This will indeed save considerable time on factoring larger
numbers, but requires that a list of prime numbers be stored or generated.
We leave this as an exercise for those desiring additional speed. The
critical statements are

```
READ   F
DATA   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,
DATA   53, 59, 61, 67, ..., etc.
DATA
```

where the DATA statements contain a list of all primes
that are $\leq \sqrt{9999999999999999}$ or < 100000000   if 16-digit arithme-
tic is being used.

The point of this exercise is <u>not</u> to illustrate how to write a factoring program, but rather to show you that it is very easy to <u>improve</u> most programs, including commerical programs written in books (even <u>this</u> one). Just because a program "works" does not mean it is a good, or even an acceptable, program. The critical questions are:

1. Does it do the job for which it was designed? (Does it work?) If not, it is useless.

2. Can I improve it noticeably <u>without much effort</u>? If so, improve it and test the new program.

3. If it requires much effort to improve the program, will the results be worth the effort? (If the program will seldom be used again, and if it does not take an unreasonable amount of computer time as it is, it probably is <u>not</u> worth the effort to improve it <u>unless</u> you want to use it to brag a bit.)

4. You should always ask yourself if the method (algorithm) used by the original programmer is the best one you can think of.

Experienced programmers have a saying that contains much truth.

Only after you have written, debugged, and tested a program, do you understand it well enough to program it. If it is a major program, after you get the program all up and running smoothly, scrap the entire program and THINK carefully about it. Then start over from scratch and write a better program.

We examine another example in detail before trying new micro-Research Problems. We consider the four-digit case first, then after that is investigated, ask you to expand it in micro-research Problem 18, as suggested in the STEPS FOR COMPUTER ASSISTED PROBLEM SOLVING.

EXAMPLE 17-2

Let ABCD be a 4-digit number such as 4721

```
100 LET X = A
    A = ABS |A-B|
    B = ABS |B-C|
    C = ABS |C-D|
    D = ABS |D-X|
    PRINT A ; B ; C ; D
    IF A+B+C+D <> 0 THEN 100 ELSE STOP
```

Your first problem is to write a BASIC program that will show that each 4-digit starting-value number will converge to 0000 in 9 or fewer steps.

Write your own program before continuing.

EXAMPLE

$ABCD = 4721$
$X = 4$
$A = |4-7| = 3$
$B = |7-2| = 5$
$C = |2-1| = 1$
$D = |1-4| = 3$

$NEW\ ABCD = 3513$
$X = 3$
$A = |3-5| = 2$
$B = |5-1| = 4$
$C = |1-3| = 2$
$D = |3-3| = 0$

$NEW\ ABCD = 2420$
$X = 2$
$A = |2-4| = 2$
$B = |4-2| = 2$
$C = |2-0| = 2$
$D = |0-2| = 2$

$NEW\ ABCD = 2222$
$X = 2$
$A = |2-2| = 0$
$B = |2-2| = 0$
$C = |2-2| = 0$
$D = |2-2| = 0$
$NEW\ ABCD = 0000$
STOP

There are several ways this could be done. One of the more obvious is:

```
21Ø FOR M = 1 TO 9
22Ø    FOR H = Ø TO 9
225       PRINT   "WORKING ON M, H = "; M; H
23Ø       FOR T = Ø TO 9
24Ø          FOR U = Ø TO 9
25Ø             A = M
26Ø             B = H
27Ø             C = T
28Ø             D = U
29Ø             PRINT  "STARTING WITH A, B, C, D = " A; B; C; D

3ØØ             N = Ø
31Ø             X = A
32Ø             N = N+1
33Ø             A = ABS(A-B)
34Ø             B = ABS(B-C)
35Ø             C = ABS(C-D)
36Ø             D = ABS(D-X)
37Ø             PRINT  A; B; C; D
38Ø             IF A + B + C + D > Ø THEN 31Ø
39Ø             PRINT M;H;T;U; "TAKES ";N; "STEPS TO CONVERGE TO ØØØØ."

4ØØ          NEXT U
41Ø       NEXT T
42Ø    NEXT H
43Ø NEXT M

5ØØ PRINT
51Ø PRINT  "FINISHED WITH 4-DIGIT TESTS."
```

This program runs through all of the four-digit starting-values, printing out new values as they are developed, and printing out the number N of steps each starting-value requires to converge to ØØØØ before going to a new starting-value whenever ABCD = ØØØØ.

Put it on your computer and run it. It is interesting to watch, but it sure takes a <u>long time</u> to test 9000 cases!

Of course, you don't really <u>need</u> to watch, since, if a given starting value ever fails to produce ØØØØ, the program will continue trying, and the change in pattern will be clear.

However, if we are <u>not</u> going to watch the program, it can be speeded up a lot by eliminating statements 29Ø and 37Ø. Try it and see. Be sure you also recognize how to tell if this revised program ever finds a starting value which does not converge to ØØØØ.

It is still a long program, and besides, it doesn't really do what we set out to do (to show that every 4-digit starting-value converges to

0000 in <u>nine</u> or fewer steps).

   Hmmm...let's store the largest N-value in another variable called L.
(Why not in B for biggest?)   We can do that by inserting:

```
205 L = 0
385 IF L > = N THEN 400
395 L = N
515 PRINT   "LARGEST N VALUE IS " ; L
```

Let's see how that works.

```
205 L = 0
210 FOR M = 1 TO 9
220    FOR H = 0 TO 9
225       PRINT  "WORKING ON M, H =  "; M; H
230       FOR T = 0 TO 9
240          FOR U = 0 TO 9
250             A = M
260             B = H
270             C = T
280             D = U

300             N = 0
310             X = A
320             N = N+1
330             A = ABS(A-B)
340             B = ABS(B-C)
350             C = ABS(C-D)
360             D = ABS(D-X)
370             PRINT A;B;C;D;
380             IF A + B + C + D > 0     THEN 310
385             IF L > = N THEN 400
390             PRINT M;H;T;U; "TAKES ";N; "STEPS TO CONVERGE TO 0000."
395             L = N

400             NEXT U
410          NEXT T
420       NEXT H
430 NEXT M

500 PRINT
510 PRINT  "FINISHED WITH 4-DIGIT TESTS."
515 PRINT  "LARGEST N VALUE IS " ; L
```

When we ran the above program, we discovered L = 8. You can modify the
program to determine all 4-digit starting-values that require 8 steps to
converge to 0000.

microRESEARCH PROBLEMS

(continued from Lesson 5)

18. a. Extend Example 17-2 to 5-digit starting-values A B C D E

$$A = X$$
$$A = ABS(A-B)$$
$$B = ABS(B-C)$$
$$C = ABS(C-D)$$
$$D = ABS(D-E)$$
$$E = ABS(E-X)$$

etc. Let's print out each set (like we did in 29∅ and 37∅) to see what is going on at the early stages of our investigation.

It may come as a shock to you that not all values converge to ∅∅∅∅∅. Indeed, apparently quite a few converge to ∅∅∅XX (where the digits XX are alike, but not necessarily ∅∅), and then enter a long loop which eventually returns to ∅∅∅XX.

b. Modify the program or write a new one to investigate what really does happen to 5-digit starting-values.

c. Extend your investigations to include 6, 7, and 8-digit starting-values and 2 and 3-digit starting-values, as well as the 4 and 5-digit values already investigated.

Would you like to conjecture that all starting-values converge to ∅∅...XX, but that we can guarantee XX = ∅∅ only if the number of digits in the starting-value is a power of 2? Why, or why not?

19. Write a program that will shuffle 52 cards in the computer and deal out 4 bridge hands, and then bid one of the hands, with human input for bids on the other three hands.

20. Write a program to create Haiku poetry.

21. Write a program that will accept seven subscripted variables V(0), V(1),...,V(6) and will test to determine whether or not any of the variables contain equal values. If all seven variables are different, print "ALL VARIABLES ARE DISTINCT." Otherwise, print the subscripts and the common value of those variables containing the same value. If there is more than one set of subscripted variables with the same values, the program should also indicate the subscripts and common values of other sets.

$$V(0) = 4$$
$$V(1) = -3$$
$$V(2) = 1.765$$
$$V(3) = 4$$
$$V(4) = -2$$
$$V(5) = -3$$
$$V(6) = 4$$

The output should be

```
SUBSCRIPTS = 0 3 6   VALUE = 4
SUBSCRIPTS = 1 5     VALUE = -3
```

22. Write a program to accept one of the three symbols from the key-
board — $\emptyset$ + . The program should then examine a string of
DATA (from DATA statements or from tape) and if

    — was depressed at the keyboard, display the number and sum of
       the negative DATA values.

    + was depressed at the keyboard, display the number and sum of
       the positive DATA values.

    $\emptyset$ was depressed at the keyboard, display the number of zeros
       in the DATA.

23. A theorem from number theory states that every integer N is
    expressible in the form

    $$N = X^2 + Y^2 - Z^2 \quad \text{where X, Y, and Z are integers.}$$

    a. Write a program that will accept values of N between -100
       and $+100$ and print out N (as the sum of two integer squares
       minus another integer square) before returning to accept an-
       other value N.

    b. Enclose your program in a FOR...NEXT loop to test all
       $-100 < N < 100$.

    c. Can you speed your program up so it could feasibly work for
       $-5000 < N < 5000$?

24. Euclid's proof that there exist infinitely many prime integers uses
    the expression

    $$N_k = 1 + (2*3*5*7*11*\ldots*P_k)$$

    where the product in the parentheses is the product of the first
    k positive primes.

    Write a computer program to determine the first 20 values of N_k.

Extend your program to determine which of these N_k values are prime and what the factors of the non-prime N_k are. (Note: not all N_k are prime, but all of N_k's prime factors are larger than P_k and less than $(1 + \sqrt{N_k}\ )$.

25. Knowing that our calendar repeats on a 400-year cycle, determine on which day of the week the 13th day of the month is most apt to occur. (Is it really Friday?)

26. Write a program to make a table showing the values of

    N N^2 R

    (Where R is the remainder when N^2 is divided by 12.)

    for N = 1, 2, 3, ..., 20. Then jump to a conclusion (call it a conjecture, if anyone asks you) and then either prove or disprove your conjecture.

27. Determine the sum of the first k terms in the series

    $1 + 22 + 333 + 4444 + ...$

    For a solution, consult page 807 (E 1405), American Mathematical Monthly Vol 67 (1960).

28. Determine all of the 4-digit integers which are perfect squares and in which all four of the digits are even. Your final program should produce the results in less than 3 seconds of TRS-80 Level II BASIC RUN time. If it takes much more than that, please do some additional analysis and try again.

29. Devise a program that will draw a dart target on the TRS-80 screen (squares are much easier to draw than circles), with numbers shown in each area (including negative numbers outside the target if you wish). Then permit a player to toss 5 darts by typing coordinates X,Y of a point. Add a little "wobble" to the point chosen by using X = X + (RND(9) − 5): Y = Y + (RND(5) −3) or some such device before displaying SET(X,Y) as a (first blinking, then permanent) pixel on the screen. Your program should also keep a running total of the number of points the player made.

    After the program is up and running--rewrite it so that the center of the target is not at the center of the screen. Also, permit the gamemaster to assign the values to each target area before the game starts.

30. The reverse of an integer (whole number) is the number obtained by writing its digits in reverse order—that is, the reverse of N=1089 is R=9801 and the reverse of N=13 is R=31.

   a) Your project is to find values of N such that N does not equal its reverse, N≠R, but the product N*R is a perfect square. Since 1089*9801 = 10673289 = $(3267)^2$ such numbers clearly exist. How many can you find?

31. The numbers 12 and 21 are reverses of one another, as are 144 and 441, since they contain the same digits, but in reverse order. Interestingly, $12^2 = 144$ and $21^2 = 441$. Find other pairs N and N´ such that N and N´ are reverses with N≠N´, and such that N^2 and $(N´)^2$ are also reverses.

32. You borrow an amount P dollars at I percent per annum interest rate on the unpaid balance. You agree to pay off M dollars per month. Write a program that will print out a table showing at the end of each month:

Amount of interest   Amount of principal   Remaining   Total Interest   Total
paid that month   paid that month   principal   paid to date   paid
                                                                         to
                                                                         date

   Note that if in the last month (principal + interest) < M you should not have to pay the full $M.

33. Lesson 4 contained a program designed to help you learn where the various pixel blocks were located on the screen. Extend the idea by first having the player type a difficulty number N where 1 is hard and 9 is easy, perhaps using

INPUT "PLEASE TYPE DIFFICULTY NUMBER BETWEEN 1 (HARD) AND 9 (EASY); N

   Then give the player 1+RND(N) chances to locate the lighted block with blinking pixel, before clearing the screen and choosing another randomly positioned lighted block.

34. Write a program to accept a positive integer K and then type out K primes of the form $N^2 + 1$.
   Example: If K = 3
           the primes are 2, 17, 37
           each of which is prime and of the form $(N^2 + 1)$.

35. Write a program containing 20 or fewer BASIC instructions without using a PRINT" (PRINT *followed by quotes*) instruction to make a caricature of a face or person or animal.

36. Design a program which will serve as a timer for a series of 10-minute speeches. When the time keeper types RUN [ENTER] your program should display 10 MINUTES LEFT on the screen in normal size letters. The number of minutes should change every minute. When the speaker gets to 1 minute left the program should display some attention getting device on the screen and the switch to double size letters and display

     0 MINUTES 59 SECONDS

           58 SECONDS

etc. until 0 SECONDS is reached, whereupon the screen should go wild (possibly including an audio blast if you are equipped for audio output).

37. Prove or disprove that, for N a positive integer,

$$N^4 + 2N^3 + 2N^2 + 1 \text{ is}$$

never a perfect square. (Don't expect the computer to do this for you.)

38. Your local Utopia Society decides to replace their weekly Bingo session with a Numbers Betting Game to be available in gas stations and grocery stores. People can place bets on any three digit number from 000 to 999. The winning number is the three digits in the thousands places of the number of shares sold on the New York Stock Exchange that day. The payoff is 500 times the amount bet for bets up to $20. For bets above $20 the payoff is $10,000 plus 100 times the amount bet.

Your first task is to write a program to simulate the betting system and come up with estimates of likely profits or losses. You should also discover (with or without using a computer)what amount of bet will produce the greatest payoff for the bettor. If all bettors wagered this amount (best possible for them, worst for the Utopians), how much would you expect the Utopia Society to make or lose on each approximately $10,000 that was bet? Assume that 10¢ of each dollar bet goes to the person taking the bet, and 15¢ of each dollar bet is other overhead.

39. Compute and print the number 2^{75} exactly.

40. Find all positive integers X,Y,Z such that

$$X^2 + Y^2 = Z^2 \text{ with } Z < 1000.$$

Compare the speed of your program with the speed of programs written by your friends.

41. a) Two rods of equal length are divided into 250 and 243 equal parts respectively. Their ends are coincident. Find the divisions which are nearest together.

b) Same problem but divide the two equal rods into M and N equal parts. Write a program to accept positive integer input M and N and display the two values which are nearest together.

42. Consider the recursive relation defined by:

Let B = The biggest number that can be made by rearranging the digits of N_k.

S = The smallest number that can be made by rearranging the digits of N_k.

$$N_{k+1} = B - S$$

a) If N_0 is a four-digit starting value, one of two things happen:

i) If all the digits of N_0 are identical, the $N_{k+1} = 0$ for all $K \geq 0$.

ii) If N_0 has at least two distinct digits, then the relation converges to $N_{k+1} = 6174$ which then repeats forever. Investigate this phenomenon.

b) If N_0 is a six-digit integer, there are 384 non-zero possible values of N_1, some of which lead to the repeater 631764, but some of which do not. Investigate other starting values N_0 having 6 digits.

c) Investigate the $N_{k+1} = $ BIG – SMALL phenomenon for starting values of other sizes.

43. It is well-known that the reciprocals of integers form repeating decimals of period p (we say, p = 0 if the expansion terminates as on 1/2 = .500; p = 1 for 1/3 = .33; etc.) Our problem is to create a table which will give the smallest positive integer N(K) such that its reciprocal has a repeating decimal expansion of length K.

44. If N_0 is a positive integer, it can be shown that the recursive relation $N_{k+1} = $ (The sum of the <u>squares</u> of the digits of N_k) will either converge to 1 or will reduce to a self-repeating cycle 37, 58, 89, 145, 42, 20, 4, 16, 37, ...

   a) Do so. Can you <u>prove</u> that every positive starting value N_0 will eventually either converge to 0,0,0, or to the cycle 20, 4, 16, 37, ...

   b) The related problem for the sum of <u>other powers</u> of the digits also merits investigation. Do so.

45. It is a common practice in mapping to approximate the area of a lake or island or other irregular shape by using an irregular polygon, whose area is easier to compute than is the area inside a curved boundary.

   Write a program into which the number N of vertices and the X and Y coordinates of the vertices (X_i, Y_i) of the polygon in consecutive order may be input and the program will then compute the polygonal area. It will be necessary to either find or devise a suitable formula. Remember this polygon is <u>not</u> a <u>regular</u> polygon since its sides may be of unequal length.

46. Given two values A and B, determine values X, Y, and Z such that

$$\frac{A}{X} = \frac{X}{Y} = \frac{Y}{Z} = \frac{Z}{B}$$

   What will your program do if A and B have different signs?

47. Liouville discovered an interesting procedure for producing sets of positive integers with the property that the sum of their cubes is equal to the square of their sum. One such set is

   $1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$. A bit of induction will show that for all N, $1^3 + 2^3 + 3^3 + ... + N^3 = (1+2+3+...+N)^2$. However, it is not known whether or not Liouville's method will produce all such sets. Write a program to produce such sets in a given range. Investigate the problem.

48. Write a program that will compute how much an investment of P dollars will be worth Y years from now if it is invested at I percent per annum interest, compounded N times per year. A useful formula is:

   $V = P*(1 + I/(100*N))^{N*Y}$ (Where V is the value after Y years.)

49. Design a program to coach emergency first-aid treatment at the "Boy Scout" level. The program should ask questions answerable by YES or NO and prescribe action based on the response.


```
                         Is
                   patient bleeding          Yes
      No              severely?

                                      Bind wounds to stop
                                      flow of blood.  Do
                                      not use a tourniquet
                                      unless life is
                                      threatened.

                         Is
      No           patient conscious?        Yes



                         Is
                 there any evidence that
      No         patient has taken poison    Yes
                 or excessive medi-
                      cations?

                              Type number to indicate likely
                              material ingested:

                                1  Prescription medicine
                                2  Other medicine
                                3  Soaps, detergents, cleansers
                                4  Insecticides, bugkillers
                                5  Caustics:  lye, Drano, bowl
                                     cleaner, Liquid Plumber
                                6  Gasoline-like products
                                7  Paints, stains, enamels
                                8  Heavy oils or grease

                              ON P GOTO ...

                         1  2  3  4  5  6  7  8
```

50. Either find a ten-digit positive integer N with each digit different, and such that N is also a perfect 10th power of an integer, or show that no such N exists.

51. Consider a large lattice of points (100 x 100 would be large in this case and a big sheet of graph paper will do nicely) with equal spacing in the x and y directions. Our real problem is to place as many points as possible on the lattice subject to the restriction that no subset of three points shall form a right triangle. Before tackling the real problem, consider the much simpler problem of a program to find and print out 6 or 8 patterns of points on the lattice which have the property that no three points form a right triangle and that the set is maximum in the sense that if one more point were added to the given arrangement, a right triangle would be formed. Possibly placing points down the diagonal has this property. Then try to answer (with proofs) the questions:

a) What is the largest number of points which may be so placed?

b) What is the smallest number of points in a maximum set?

c) Restate the problem so that using every point of two adjacent sides of the square excepting the corner point is not a solution and solve the new problem.

52. Write a program to sort a set of 25 names alphabetically. Ponder this one. Sorting represents one of the most time-consuming uses of modern computers. Consider sorting and entering the approximately 100,000 grades received by 20,000 students each term at a major university.

53. J. Maxfield has shown (Math Magazine, Vol. 43, #2 March, 1970, pp.64-67) that given a sequence of digits K = $d_1 d_2 d_3 \ldots d_n$, there exists an integer N such that N! begins with the sequence of digits K. Write a program to create a table such that the smallest such positive integer N(K) is determined for each K ≤ 100. You may also wish to print out the floating point value of N! if it will be available at no additional cost. Eventually, we would like a formula for N(K) = smallest positive integer N such that N! begins with the sequence of digits K, but that may be too much to hope for. Your table might well resemble the one below:

K	N(K)	N!
1	1	1
2	2	2
3	9	362880
4	8	40320
5	7	5040
6	3	6
7	6	720
⋮	⋮	⋮

54. Mathematicians have proved that there exist arbitrarily long strings of consecutive positive integers such that each of the consecutive integers has as a factor, a perfect square greater than one (probably different squares for different integers). This may be restated as follows:

   Given a positive integer N, there exist strings of N consecutive positive integers each of which contains a factor which is a perfect square greater than one.

   If N = 3   48, 49, 50 contain 4^2, 7^2, 5^2 as factors.
                98, 99, 100 contain 7^2, 3^2, 10^2 as factors.

   If N = 4   242, 243, 244, 245 contain 11^2, 3^2, 2^2, 7^2 factors.

   If N = 7   217070, 217071, 217072, 217073, 217074, 217075, 217076
   contain as
   factors 7^2 3^2 2^2 113^2 11^2 5^2 4^2

   Your problem is to find the first (i.e., involving smallest integers) string of N consecutive integers each of which contains a square >1 as a factor for N = 2, 3, ..., 10.

55. This one is tough! Mathematicians have shown a similar theorem to that given in Problem 54 holds with "perfect square" replaced by "perfect cube" or even "perfect Kth power". Investigate this for K = 3, 4, 5, and N = 2, 3, ..., 10. (Or even N = 2, 3, 4, 5, 6, for starters.)

   Examples: 3rd Powers (K = 3)   4th Powers (K = 4)

   N=2   80   81   N=2   80   81
            8   27   16   81

   N=3   1375   1376 1377   N=3   33614 33615 33616
            125   8   27   2401   81   16

   N=4   22624 22625 22626 22627
              8   125   27 1331

56. Find a set of distinct positive integers, each of which is less than 100, such that it satisfies both of the following conditions.

   a) No combination (or subset) of them added together will total 100.

   b) The set contains as many distinct, positive integers as possible.

   Hint: You may be able to solve this without using a computer.

57. The square of the number 36363636364 has the unusual property (in base 10) that the second half of the square is an exact duplicate of the digits in the first half

$$(36363636364)^2 = 13223140496,13223140496$$

as shown by the central comma above.

Such periodic squares also exist in bases other than 10. For example, in base 4,
$$(21213_4)^2 = 11301,11301_4 = 378225_{10} = (615_{10})^2$$

Your problem is to write a program that will examine perfect squares, expressed in some base $B > 1$, (use base B = 10, if you wish) and find perfect squares whose expression in base B is periodic (period 2 or more). See <u>Mathematics Magazine</u>, March 1975, p. 97.

58. An integer (whole number) greater than 1 is said to be <u>prime</u> if its only positive factors are itself and one. There are many 4 x 4 arrays of digits such that each row and each column is a 4-digit prime.

Example:

6	1	8	9	Where	6189	6359
3	0	2	3		3023	1021
5	2	7	3		5273	8273
9	1	3	7		9137	9337 are each prime.

a) Find some additional such arrays. Then, hunt for arrays whose main (upper left to lower right) diagonal also contains a prime.

b) What happens if you try to find 5 x 5 arrays containing 5-digit primes in each row, each column and main diagonal?

59. Write a program to plot a graph $Y = f(X)$.

Your program may request the user to put the function in a certain statement. It should ask the user for what range of X-values he wishes the graph plotted and then compute (or ask the user) for the largest and smallest Y-value that will be used. It should then scale the function suitably for display on the CRT screen. Your program should also display for the user (perhaps using PRINT @ 960, "   ") the X-range and the Y-range covered by the graph.

60. Find the smallest positive integer M such that

$$N = 7*10^M + 1 \qquad \text{is \underline{not prime}.}$$

61. Find and print all the five-digit positive integers which contain exactly the same digits as their trebles. The leading digit must not be zero.

    Example: 1ØØ35 * 3 = 3Ø1Ø5

    Please also record the computer time used on your final run. Compare with that of other TRS-80 users you know.

62. Many people from Archbishop John Tillotson (c. 1650) through Kurt Vonnegut (1950) and beyond have toyed with the idea that if enough monkeys were permitted to pound typewriters for long enough, all the great classics of literature would eventually be reproduced. We can easily improve the chances of something legible being produced by picking keys in the approximate order of their frequency in normal English rather than at random. Write a program that will print out the following symbols, with the given approximate frequency per 1000. To simplify the problem we restrict our selection of punctuation to "space", and have omitted V, K, Q, X, J, and Z, which have very low frequency in English.

KEY	space	E	T	A	O	N	I	R	S	H
FREQUENCY/1000	223	100	77	62	62	54	54	54	46	38

KEY	D	L	F	C	M	U	G	Y	P	B
FREQUENCY/1000	31	23	23	23	23	15	15	15	15	15

You may be able to further improve the likelihood of obtaining at least occasional words by including digraphs (two-letter groups) as well as single letters. English digraph frequencies are approximately

DIGRAPH	TH	HE	ER	AN	IN	ON	RE	AT	ED	ST	ND	ES
FREQUENCY/1000	38	31	18	17	17	16	14	14	13	12	12	12

Additional frequency data for English, French, Spanish, German, and Italian are available in Sophisticated Ciphers, available from Mu Alpha Theta Math Club, Room 423, 601 Elm street, Norman, OK, 73019, or at your library.

63. Find all n-digit positive integers in which the n-digit number is an exact n-th power of an integer.

    Examples: $81 = 9^2$ $125 = 5^3$

64. Two positive integers are amicable (friendly) if the proper (including one, but not including the number itself) divisors of each add up to the other. 220 has divisors 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, whose sum is 284. 284 has divisors 1, 2, 4, 71, 142, whose sum is 220, thus 220 and 284 are said to be amicable. Some other amicable pairs are (17296, 18416) and (1184, 1210). Write a program that will find all amicable pairs between two given bounds S and B and which will also keep track of any number which has a divisor sum greater than B, so these can be used in a future run with (new S) = (old B) and a larger B. Your final program should also account for any "carry over" values from previous runs.

65. Amicable pairs (problem 64) have been generalized to amicable chains (sociable chains) in which the sum of the proper divisors of each number in the chain is the next number and the last member of the chain has the first member as the sum of its divisors. The only chains generally known are the two four-link chains (found in 1965):

(2115324, 3317740, 3649556, 2797612) and
(1264460, 1547860, 1727636, 1305184),

the five-link chain:

(12496, 14288, 15472, 14536, 14264),

and a stupendous 28-link chain which contains the number 14316. However, there are others.

   a) Write a program to investigate amicable chains containing 50 or fewer links.

   b) What happens to chains that do not loop back to the starting value?

66. Write a program that will match up 50 men and 50 women as dance partners. Your first attempt can use numbers, if you wish, but the final result should be two lists; one with the men's names in alphabetical order and showing the randomly matched partner's name, the other with the womens' names in alphabetical order and showing the name of each woman's partner (hopefully, the same matching as above). Each person should be matched with one and only one partner and that partner should be of the opposite sex.

67. a) Write a computer program to accept three values M, D, Y and determine whether or not they are acceptable values for Month, Day, Year, in that order.

   b) Extend the above program to accept two sets of three numbers and if both are acceptable as Month, Day, Year values, determine the number of days between them, including both dates in your count. Example: 2, 20, 1978 to 2, 22, 1978 is 3 days.

68. A computer is apt to obtain different values for
F(N) = 1*2*3*...(N-1)*N and for G(N) = N*(N-1)*...3*2*1.
Furthermore, the difference is apt to be non-trivial. For N = 30,
the difference can be greater than a quintillion (and that's more
than the number of cents in the national debt). Investigate this
phenomenon, using a program of your own devising, or using:

```
100 FOR N = 10 TO 50
110     F = 1
120     G = N
130     FOR K = 2 TO N
140         F = F*K
150         G = G*(N-K+1)
160     NEXT K
170     PRINT N, F; G, ABS(F-G)
180 NEXT N
```

69. Mathematicians have shown that given a sequence of digits
$S = d_1 d_2 d_3 ... d_n$ there exists an integer N such that 2^N begins with
the sequence S. (As a matter of fact, for 2 we may substitute any
positive integer which is not a power of 10, i.e., not $10^0=1$, $10^1=10$,
$10^2=100$, etc.)

Your problem is to produce a table giving S, N, 2^N for S = 1 to 100
such that N(S) is the smallest positive integer such that $2^{N(S)}$
begins with S. I think the first few values are:

S	Smallest N>0	2^N
1	4	16
2	1	2
3	5	32
4	2	4
5	9	512
6	6	64
7	46	70368744177664
8	3	8
9	53	etc.
10	10	
11	50	

70. Problem 11 (Lesson 5) asks you to generate and print out in increas-
ing order of size, the positive integers of the form $N = 2^k \cdot 3^m$
where $k \geq 0$ and $m \geq 0$. Generalize this to integers of the form
$A^k \cdot B^m$ where A and B are read using an INPUT statement.
Note that A and B need not be prime. Your program should work if
A = 6 and B = 10.

71. Use an array containing the proper fractions in lowest terms to establish a "position number" for each entry. In the array each entry in row k shall have (k+1) as denominator and the numerators in a given row shall be in increasing order.

1/2

1/3   2/3

1/4   3/4

1/5   2/5   3/5   4/5

1/6   5/6

1/7   2/7   3/7   4/7   5/7   6/7

1/8   3/8   5/8   7/8

1/9   etc.

1/1∅ etc.

.
.
.

Since only those proper fractions in lowest terms are included, the rows are not of easily predictable length.
Establish a position number N(α) for each fraction α in the array by simply counting the entries, row after row, from left to right until α is reached.

Thus   N(1/2) = 1
        N(1/3) = 2
        N(1/4) = 4
        N(2/3) = 3
        N(3/4) = 5

and so forth. In this fashion one establishes that

        N(1/13) = 46
        N(15/16) = 79
        N(1/100) = 3004
        N(3/500) = 75917
        N(7/1240) = 467048   (or do they?)

The challenge is to determine formulae or an algorithm that will enable you to do either (or both) of the following

   1. Given a proper fraction α in lowest terms, determine N(α).
   2. Given an integer K, determine the corresponding proper fraction α such that K = N(α). Use a computer to help if you can.

72. Program a game of your own choice on the computer.

73. The problem of placing eight queens on an 8x8 chess board so that no queen can attack any other queen (queens attack any piece they can reach by moving in a straight line horizontally, vertically, or on either diagonal) is an old one. You may have considered the problem before. You will find it discussed on p. 161-2 of W. W. Rouse Ball's interesting book, Mathematical Recreations & Essays or in Chapter 10 of M. Kraitchik's admirable book, Mathematical Recreations, both of which should be available at your local library.

You may write a program to solve the problem if you wish, but the problem proposed here is to examine the following program and see if you can figure out how it works and why it produces all 92 possible solutions to the eight queens program.

```
  2 CLS
  4 PRINT "     EIGHT QUEENS PROBLEM    (RUNNING TIME ABOUT 4Ø MINUTES)"
  6 PRINT
  8 PRINT "         1   5   8   6   3   7   2   4  MEANS"
 1Ø PRINT " THE QUEEN IN THE FIRST COLUMN IS ON ROW 1;"
 12 PRINT "   THE QUEEN IN THE SECOND COLUMN IS ON ROW 5;"
 14 PRINT "    THE QUEEN IN THE THIRD COLUMN IS ON ROW 8;ETC."
 16 PRINT
 2Ø DIM R(8)
 5Ø N = Ø
1ØØ C=Ø
2ØØ C=C+1
3ØØ R(C)=Ø
4ØØ IF R(C) < 8  THEN 6ØØ
5ØØ C=C -1
51Ø IF C = Ø  THEN PRINT"END OF SEARCH":STOP
52Ø GOTO 4ØØ
6ØØ R(C) = R(C) + 1
6Ø5 IF C < 2 THEN 66Ø
61Ø FOR  C1 = 1 TO C -1
62Ø     IF R(C)=R(C1) OR C+R(C)=C1+R(C1) OR R(C)-C=R(C1)-C1 THEN 4ØØ
65Ø NEXT C1
66Ø IF C < 8 THEN 2ØØ
69Ø N=N+1:PRINT"SOLUTION#";N;"=",
7ØØ FOR C1=1 TO 8
71Ø     PRINT R(C1);"   ";
72Ø NEXT C1
73Ø PRINT
74Ø GOTO 5ØØ
```

74. If you are seriously interested in more advanced computing, investigate one of the following languages for your TRS-80.

"C" or "tinyC"
"PASCAL"
"Z-80" chip language (enter from BASIC II through USR( ) function)
PILOT (a language designed for Computer Assisted Instruction)

75. Study a program written by someone else, and then improve it.

A FINAL WORD from your authors

Good Programming Practices

There is no such thing as the correct programming style. Entire books have been written on the subject, but even the most skillful practitioners of the programming art disagree on what is best. However, there are several common sense principles that merit consideration. Even these are not immutable rules, but it is prudent to follow the suggestions unless you have a good reason for not doing so on a given program. Here goes:

1. Your program should, in general, read from top to bottom in a logical fashion, except for subroutines which usually are listed after the completed program. Block the various fragments into different 100's in statement numbers. Don't feel compelled to use every tenth statement number.

2. Use occasional REM statements to explain what is happening. If you must use a GOTO statement in the heart of your program, include a REM statement explaining why.
   Your programs will have a neater appearance, and be easier to debug if you place five blank spaces between REM and the start of your remark. Try it, you'll like it.

3. Whenever you use a FOR-NEXT loop, indent the instructions between FOR and NEXT three spaces. If you use nested FOR-NEXT loops, stair-step them. It makes programs much easier to understand.

4. Use the faster forms whenever feasible.
   (See Lesson 11.)

5. Get to know your computer. Become thoroughly familiar with the quirks and excentricities of your computer and of the BASIC (or other) language you are using.

Above all:
   Don't let these or any other rules spoil your fun---but do remember that well-written, easy-to-follow programs are no harder to think or write than sloppy kluges and are much more profitable in use and maintenance. Think of a computer program as a living, growing, ever-changing thing, not as a finished, static object.

HAVE FUN

Josephine Andree
Richard V. Andree, 1980

A

B

C

D

E

Subscripted Variables, 146
Symbol >_ , 5, 12, 41
SYSTEM, 184

Special Characters

'	Abbreviation for :REM
%	Makes variable integer-precision.
!	Makes variable single-precision.
#	Makes variable double-precision.
$	Makes variable string type.
:	Separates statements on the same line.
?	Same as PRINT (but L? can't be substituted for LPRINT).
,	PRINT punctuation: spaces over to the next 16-column PRINT zone.
;	PRINT punctuation: separates items in a PRINT list but does not add spaces when they are output.

Error Messages

Abbreviation	Explanation
NF	NEXT without FOR
SN	Syntax error
RG	RETURN without GOSUB
OD	Out of data
FC	Illegal function call
OV	Overflow
OM	Out of memory
UL	Undefined line
BS	Subscript out of range
DD	Redimensioned array
0	Division by zero
ID	Illegal direct
TM	Type mismatch
OS	Out of string space
LS	String too long
ST	String formula too complex
CN	Can't continue
NR	No RESUME
RW	RESUME without error
UE	Undefined error
MO	Missing operand
FD	Bad file data
L3	Disk BASIC feature

Edit Commands

EDIT	Puts computer into edit mode for specified **Edit Commands.**
EDIT 100	EDIT.

A	Cancels changes and starts again.
■C	Changes n characters.
■D	Deletes n characters.
E	Ends editing and saves all changes.
H	Hacks line and inserts at end.
I	Inserts characters.
■K c	Kills all characters up to nth occurrence of c
L	Lists the line.
Q	Quits edit mode and cancels all changes.
■S c	Searches for nth occurrence of c.
X	Extends line (inserts at end).
SHIFT ↑	Causes escape from command.
ENTER	Records all changes and exits edit mode.
■ SPACEBAR	Moves cursor n spaces to the right.
■ ←	Moves cursor n spaces to the left.

Control Keys

←	Cancels last character typed; moves cursor back one space.
SHIFT ←	Erases current line.
BREAK	Interrupts anything in progress and returns to command level.
CLEAR	Clears the screen.
ENTER	Signifies end of current line.
SPACEBAR	Enters a space (blank) character and moves cursor one space forward.
→	Advances cursor to next tab position.
SHIFT →	Puts display in 32-character mode.
↓	Line feed and carriage return.
SHIFT ↓	"Control" key—hold down these two and press any key A-Z for control A control Z.
SHIFT ↓ ·	Copies the display contents to the printer.
SHIFT @	Causes currently executing program to pause (press any key to continue).